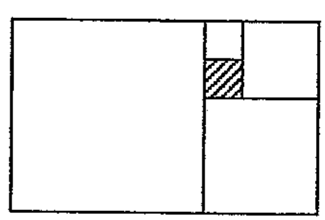
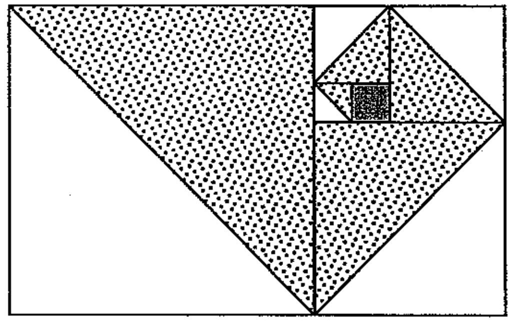
Exercises 8.1

1. In the figure, the shaded region is a square 1 unit on a side. The other four pieces are also squares. What is the total area that is not shaded?



2. In the figure, the gray region is a square of side 1. Each of the other regions is also a square. What is the area of the speckled region?

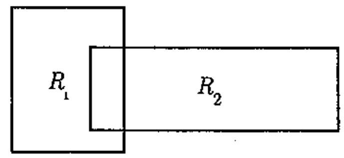


3. In the figures, R_1 and R_2 are rectangles. $a(R_1) = 20$ and $a(R_2) = 30$. Region R is the union of regions R_1 and R_2.

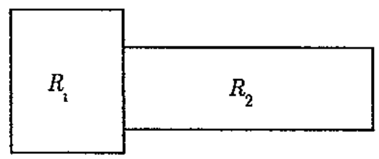
   a. Does $a(R) = 50$? Why or why not?

   b. Does $a(R) = 50$? Why or why not?

Part a.

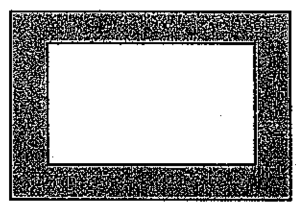


Part b.

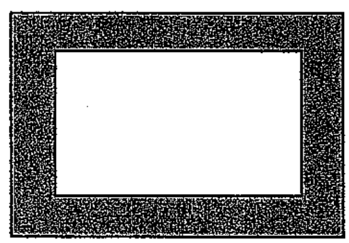


4. One store sells a certain carpet at \$3.50 per square foot. A second store sells the same carpet for \$30.00 per square yard. Which is the better buy?

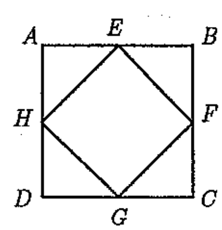
5. A rectangular picture 7 inches by 10 inches has a matting $1\frac{1}{2}$ inches wide surrounding it (the shaded part of the diagram). What is the area of the matting?



6. A rectangular picture 8 inches by 12 inches has a matting x inches wide surrounding it (the shaded part of the diagram). If the area of the matting is 44 square inches, then what is the value of x?



7. Square $ABCD$ has a side of $2a$. E, F, G, and H are the midpoints of the sides of $ABCD$. What is the area of $EFGH$?

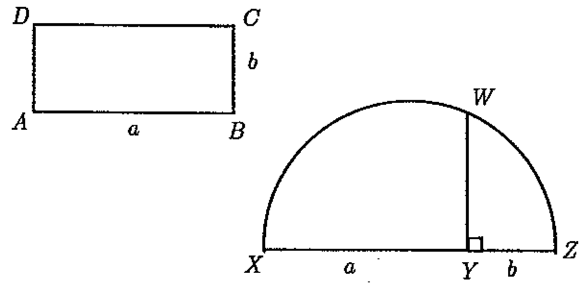


8. The diagonal of a rectangle is $\sqrt{13}$ and one side is 2. What is the area of the rectangle?

9. The sides of a rectangle are in the ratio 3 to 4. The area of the rectangle is 108 square inches. What are the dimensions of the rectangle?

10. What is the side of a square that has the same area as a 3 by 12 rectangle?

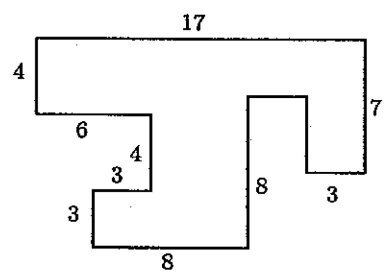
11. Rectangle $ABCD$ has $AB = a$ and $BC = b$. A semicircle is drawn on diameter $\overline{XYZ}$, where $XY = a$ and $YZ = b$. A perpendicular is drawn to $\overline{XYZ}$ at Y, intersecting the semicircle at W. Prove that a square with side $\overline{YW}$ has the same area as $ABCD$.



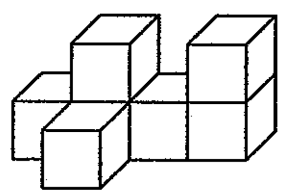
12. A rectangular lot is bounded on four sides by a path of uniform width. The dimensions of the lot inside the path are 20 yards by 24 yards. If the area of the path is 477 square yards, what is the width of the path?

13. In determining the appropriate rug size for a 12-foot by 20-foot room, a woman decides that she wishes the rug to cover three-quarters the area of the floor and have its sides equidistant from the walls. What are the dimensions of the rug she should buy?

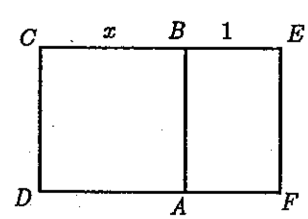
14. In the figure to the right, all angles are right angles. What is the area of the polygon?



15. A stack of cubes is glued together as shown, with five cubes on the lower layer and two on the top. The edge of each cube is 2 inches. What is the total surface area exposed to the air (including the bottom of the pile)?

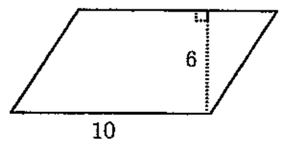


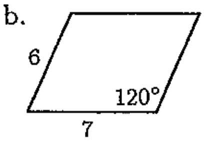
16. In the figure, $ABCD$ is a square and rectangle $ABEF$ is similar to rectangle $CDFE$. Determine $a(CDFE)$ in simplest radical form.

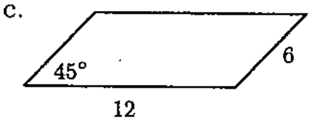


Exercises 8.2

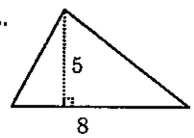
1. Determine the area of each of the parallelograms pictured below.

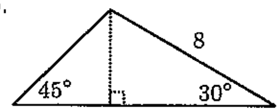
a.


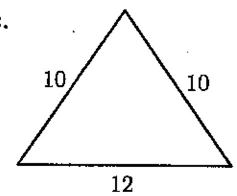
b.


c.


2. Determine the area of each of the triangles pictured below.

a.


b.


c.


3. a. Determine the area of an equilateral triangle of side 8.
   b. Determine the area of an equilateral triangle of side s.

4. a. Determine the area of a 45-45-90 triangle with hypotenuse 6.
   b. Determine the area of a 45-45-90 triangle inscribed in a circle of radius 10.

5. a. Determine the area of a 30-60-90 triangle inscribed in a circle of radius 8.
   b. Determine the area of a 30-60-90 triangle inscribed in a circle of radius r.

6. a. Determine the area of an equilateral triangle inscribed in a circle of radius 4.
   b. Determine the area of an equilateral triangle inscribed in a circle of radius $2\sqrt{3}$.
   c. Determine the area of an equilateral triangle inscribed in a circle of radius r.

7. Let ΔABC have sides of lengths a and b including an angle with measure θ. Let ΔXYZ have two sides of lengths a and b including an angle with measure $180-\theta$. Show that $a(\Delta ABC) = a(\Delta XYZ)$.

8. Find the area of a rhombus of side 12 if the larger angles are twice as big as the smaller ones.

9. Use "area equals area" to determine the radius of the circle inscribed in a 3–4–5 triangle. Compare your result with that found in the example on page 7-8.

10. Determine the radius of the inscribed circle in a 5–12–13 triangle.

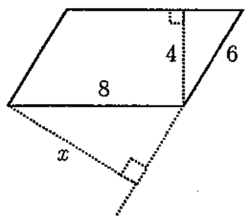
11. An isosceles triangle has an area of 24 and a base of 8.
    a. How long are the congruent sides?
    b. What is the length of the altitude to one of the legs?

12. A triangle has sides that are consecutive integers. The altitude drawn to the middle-sized side is 1 unit shorter than the shortest of the three sides. The area of the triangle is 84. What are the lengths of the three sides?

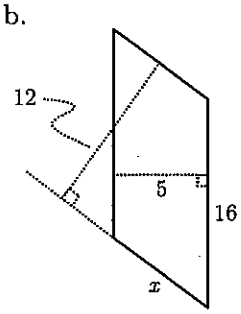
13. An isosceles triangle has legs that area twice as long as the base. What is the ratio $\frac{\text{altitude to base}}{\text{altitude to leg}}$?

14. Determine the value of x in each of the following parallelograms.

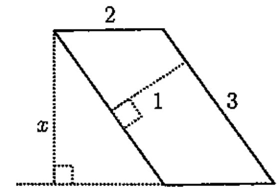
a.



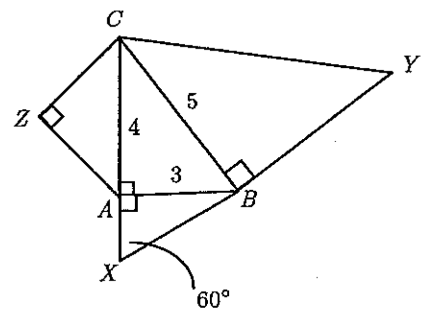
b.



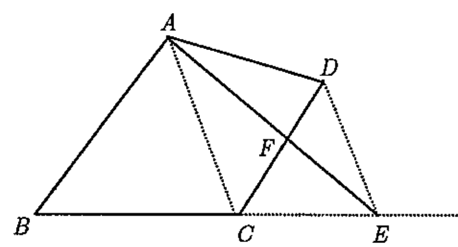
c.



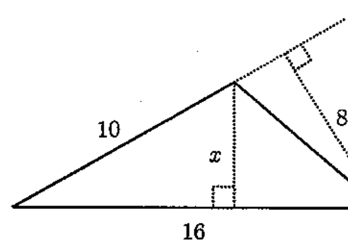
15. In the figure, ΔABC has sides of 3, 4, and 5, as indicated.
ΔAXB is a 30-60-90 triangle.
ΔBYC is a 45-45-90 triangle.
ΔCZA is a 45-45-90 triangle.
   a. Are X, B, and Y collinear? Explain.
   b. What is the perimeter of $AXBYCZ$?
   c. What is the area of $AXBYCZ$?



16. In the figure, $\overline{DE} \parallel \overline{AC}$.
Explain why $a(ABCD) = a(\Delta ABE)$.



17. Determine x in the figure to the right.



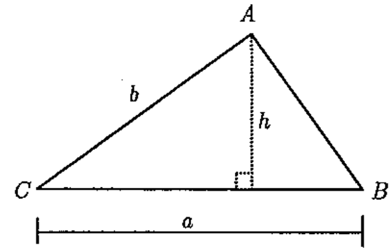
18. The diagonals of parallelogram $ABCD$ intersect at P. Prove that the four small triangles formed (ΔABP, ΔBCP, ΔCDP, and ΔDAP) have equal areas.

19. What is the length of an altitude drawn to one of the congruent sides of an isosceles triangle having sides 10, 10, and 6?

20. The altitude of a triangle is 2 inches longer than the base. The area is 24 square inches. What is the length of the base?

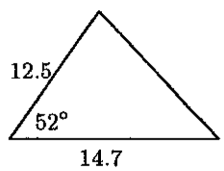
21. A parallelogram has one side twice as long as another. Those sides include a 120° angle. If the area of the parallelogram is $12\sqrt{3}$, then what is the perimeter of the parallelogram?

22. Consider ΔABC as labeled to the right.
   a. What is h in terms of b and a trigonometric function of $\angle C$?
   b. Use your result from part (a) to obtain a formula for $a(\Delta ABC)$ when two sides and the included angle are known.

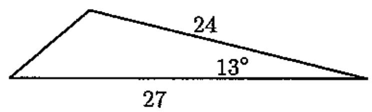


23. Determine the area (to the nearest 0.1) of each of the triangles pictured below.

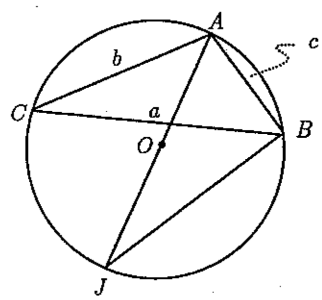
a.



b.



24. ΔABC is inscribed in $\odot O$ of radius R, and $\angle C$ is an acute angle.
   a. Using right ΔABJ, show that $\sin \angle C = \frac{c}{2R}$.
   b. Combine part 21a. with the result of problem 19 to show that $a(\Delta ABC) = \frac{abc}{4R}$.



25. Prove: If diagonal $\overline{AC}$ of quadrilateral $ABCD$ is drawn, and if $a(\Delta ABC) = a(\Delta ACD)$, then $\overline{AC}$ bisects $\overline{BD}$.

26. Point P is any point inside parallelogram $ABCD$. Prove that $a(\Delta ABP) + a(\Delta CDP) = a(\Delta BCP) + a(\Delta ADP)$.

27. An isosceles triangle has congruent sides of length 25 and an area of 168. Determine the lengths of the base and the altitude to the base.

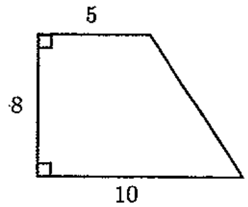
28. Let P be any point in the interior of equilateral ΔABC. Let $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ be the segments drawn from P perpendicular to the sides of ΔABC. Show that $PX + PY + PZ$ is the same for every location of point P.

29. Given equilateral ΔABC. Let P be any point lying in the interior of $\angle A$, but in the exterior of the triangle. Let $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ be the segments drawn from P perpendicular to the sides of ΔABC — extended, if necessary — with $\overline{PZ} \perp \overline{BC}$. Show that $PX + PY - PZ$ is the same for every location of the point P.

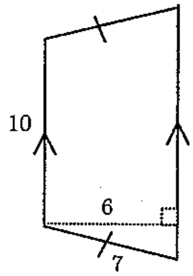
Exercises 8.3

1. Determine the area of each of the following trapezoids.

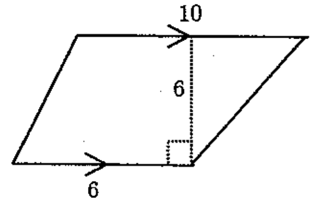
a.



b.



c.

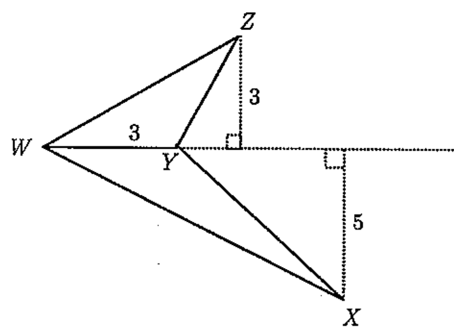


2. What is the area of a rhombus with diagonals 6 and 12?

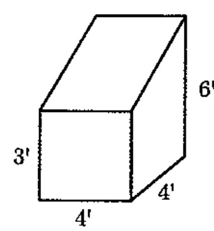
3. What is the area of a rhombus with sides of length 8 and having two 135° angles?

4. What is the area of a kite having two sides of length 5, two sides of length $\sqrt{58}$, and a short diagonal of length 6?

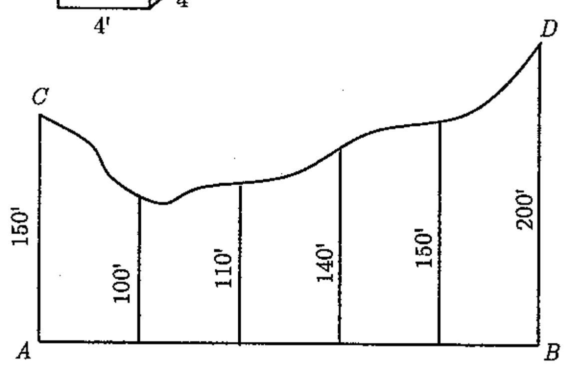
5. What is the area of the non-convex quadrilateral $WXYZ$ shown?



6. A storage bin, with the dimensions given in the figure, is to have all of its surfaces, including the lid and the bottom, painted both inside and out. If a quart of paint covers 50 square feet, then how many quarts should be purchased?



7. A man buys an irregularly shaped lot bordered by a river (C to D in the figure) and a straight road (A to B). In order to estimate the area of the parcel of land, he takes several measurements perpendicular to the road at 50-foot intervals, as shown. What is the approximate area of the piece of land?

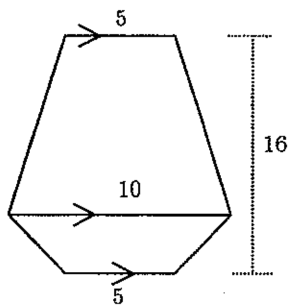


8. What is the area of a trapezoid if its altitude is 6 and its median is 8?

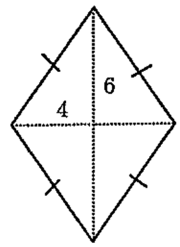
9. One base of a trapezoid is 3 longer than the other. The altitude is 4 and the area is 140. What are the lengths of the bases?

10. Find the total area of each figure pictured (in part c, do the shaded region).

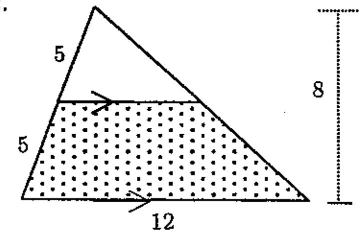
a.



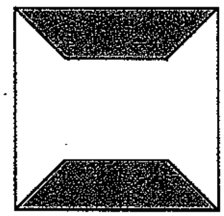
b.



c.

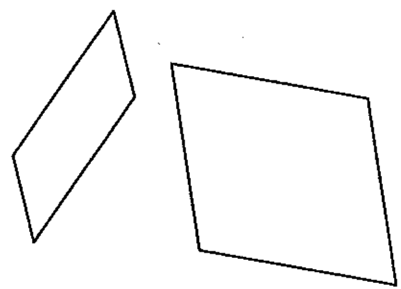


11. In the figure to the right, the outer shape is a square of side 12. The shaded regions are trapezoids with 45° base angles and legs of length $3\sqrt{2}$. What is the area of the region that is not shaded?

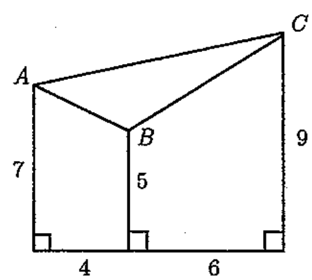


12. Prove that any line drawn through the intersection of the diagonals of a parallelogram divides the parallelogram into two regions of equal area.

13. To the right are two parallelograms. Can you draw a line that will cut the total area enclosed by the parallelograms in half? (If the total area of the two parallelograms is A, then $\frac{1}{2}A$ should be the area on one side of the line, and $\frac{1}{2}A$ the area on the other side of the line.)

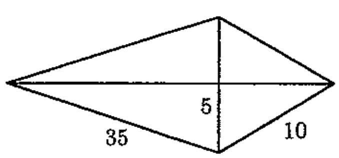


14. a. Determine the area of ΔABC.
    b. Determine the perimeter of ΔABC.

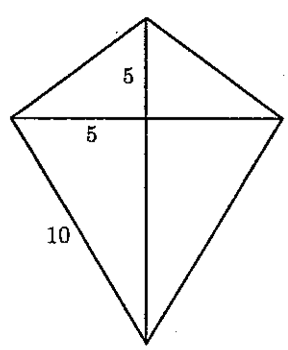


15. Determine the area of the kites shown below.

a.



b.



Exercises 8.4

1. Consider a square inscribed in a circle of radius 6.
   a. What is the apothem of the square?
   b. What is the area of the square?

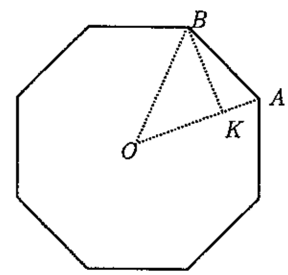
2. A circle is circumscribed about a regular hexagon having an apothem of 3.
   a. What is the radius of the hexagon?
   b. What is the area of the hexagon?

3. A regular hexagon is inscribed in a circle of radius 6, and a second regular hexagon is circumscribed about the same circle. What is the difference of the areas of the two hexagons?

4. A square is inscribed in a circle of radius R and a second square is circumscribed about the same circle. What is the ratio of the areas of the two squares?

5. Find the apothem of a regular polygon whose area is 64 and whose perimeter is 32.

6. Pictured is a regular octagon of radius 6.
   a. Compute the length AB.
   b. Compute the apothem of the octagon.
   c. Compute the area of the octagon.



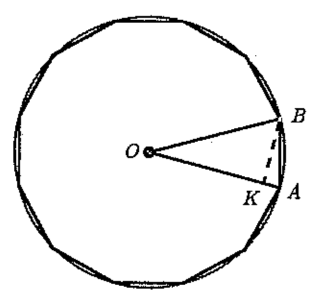
7. What is the area of a regular dodecagon inscribed in a circle of radius R?

8. What is the area of a regular octagon inscribed in a circle of radius R?

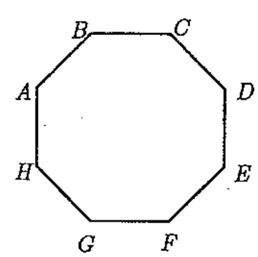
9. Equilateral triangles are inscribed in and circumscribed about a circle of radius R. What is the ratio of their areas?

10. A square and a regular hexagon are inscribed in the same circle. What is the ratio of their areas?

11. Pictured is a regular dodecagon (a polygon with 12 sides) of radius 6.
    a. What is the measure of central $\angle AOB$?
    b. What is the length OK?
    c. Compute the length AB.
    d. Compute the apothem of the dodecagon.



12. The side of the regular octagon $ABCDEFGH$ is 4. Determine each of the following exactly, if possible.
    a. The measure of an exterior angle of the octagon
    b. The length of diagonal $\overline{AD}$
    c. The length of diagonal $\overline{AC}$



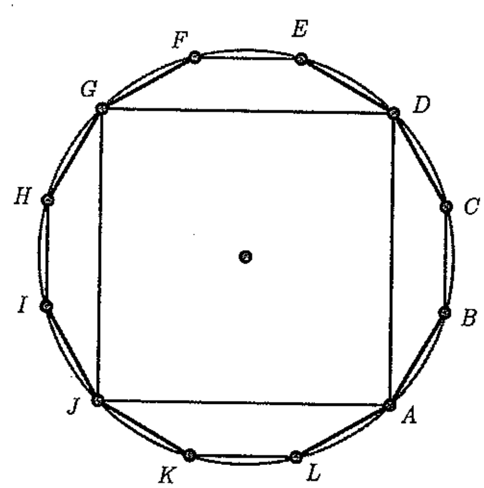
13. A regular nonagon (9-sided polygon) has a radius of 12.
   a. What is the central angle determined by one side?
   b. What is the area of the polygon (to the nearest 0.1)?
   c. What is the apothem of the polygon (to the nearest 0.1)?

14. Consider a regular polygon of 120 sides and having a radius 1.
   a. What is each of the central angles of the 120-gon?
   b. Proceeding as in the last example of this section, and using a trigonometric ratio, what is the area of one of the central triangles that make up the polygon?
   c. What is the area of the polygon?
   d. What is the area if the polygon has 1200 sides instead of only 120?
   e. What is the result if the polygon has 1,200,000 sides?

15. A regular hexagon and an equilateral triangle are both inscribed in a circle of radius r. What is $\frac{a(\text{triangle})}{a(\text{hexagon})}$?

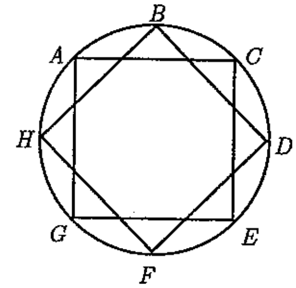
16. A square and a regular octagon are both inscribed in a circle of radius r. What is $\frac{a(\text{square})}{a(\text{octagon})}$?

17. Pictured to the right is a regular dodecagon (12-sided polygon) of side 2. $ADGJ$ is a square.
   a. What is $m\angle BAD$?
   b. What is the length of chord $\overline{AD}$?
   c. What is the area of trapezoid $ABCD$?
   d. What is the area of square $ADGJ$?
   e. What is the radius of the circle?



18. A regular hexagon is inscribed in a circle of radius r and a second regular hexagon is circumscribed about the same circle. What is the ratio of the areas of the two hexagons?

19. In the figure to the right, the octagon $ABCDEFGH$ (which is **not** drawn) is regular. Alternate vertices of that octagon are joined to form two overlapping squares. If the radius of the circle is 6, then what is the total area enclosed by the two squares?



Exercises 8.5

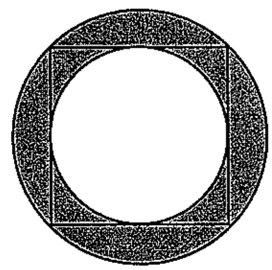
1. Determine the area of a circle of radius 3.

2. A circle has an area of 16π. What is the circumference of the circle?

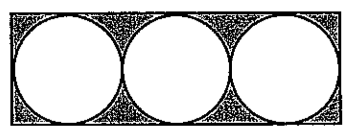
3. What is the radius of a circle whose circumference is:
   a. 6π b. 9π c. 10 d. 2

4. The apothem of a regular hexagon is $2\sqrt{3}$. Find the circumferences of the inscribed and circumscribed circles.

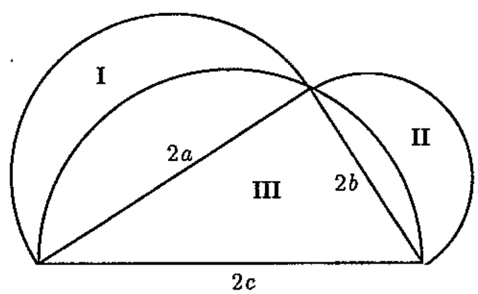
5. A square of side 6 has both inscribed and circumscribed circles. What is the area of the band between the circles?



6. Determine the area of the shaded region in terms of r, the radius of each of the three circles.

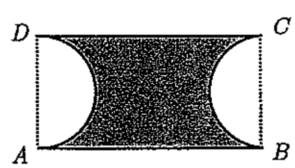


7. To the right is pictured a right triangle inscribed in a semicircle. The sides of the triangle are $2a$, $2b$, and $2c$, as shown. Semicircles are then constructed on the legs of the right triangle, forming five regions. Show that the sum of the areas of regions I and II equals the area of the triangle III.

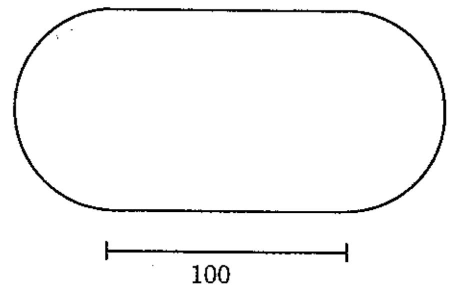


8. The area of a circle is numerically equal to the sum of the circle's circumference and the area of the inscribed square. What is the radius of the circle?

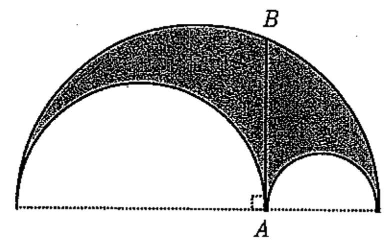
9. $ABCD$ is a rectangle, with $AB = 12$ and $BC = 6$. Semicircles are taken off of the two ends.
   a. What is the perimeter of the shaded region?
   b. What is the area of the shaded region?



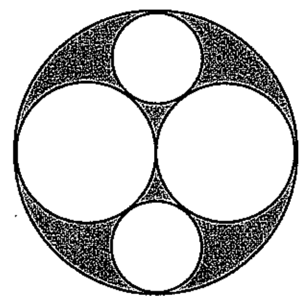
10. A 400-meter track is designed with 100-meter straight-aways and semicircular ends, as pictured to the right.
   a. What should the radius of the semicircular ends be?
   b. What is the area enclosed by the track?



11. Archimedes considered the problem of the "$\alpha\rho\beta\varepsilon\lambda o\varsigma$" — the "arbelos," or shoemaker's knife. This is formed by three semicircles with their diameters all on the same line. If the radii of the two smaller semicircles are R and r, show that the area of the arbelos is the same as the area of a circle with diameter $\overline{AB}$.

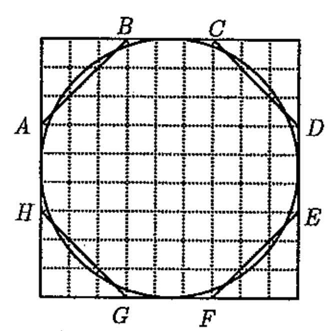


12. In the figure, the circles are tangent to each other. The large circle has a radius of 1 and the two larger inner circles have a radius of $\frac{1}{2}$.
   a. Show that the smaller circles have radius $\frac{1}{3}$.
   b. Determine the area of the shaded region.



13. If you compare the ancient Egyptian approximate formula for the area of a circle, namely $A = \left(\frac{8}{9}d\right)^2$, with the formula $a = \pi r^2$, what approximation for the value of π is obtained?

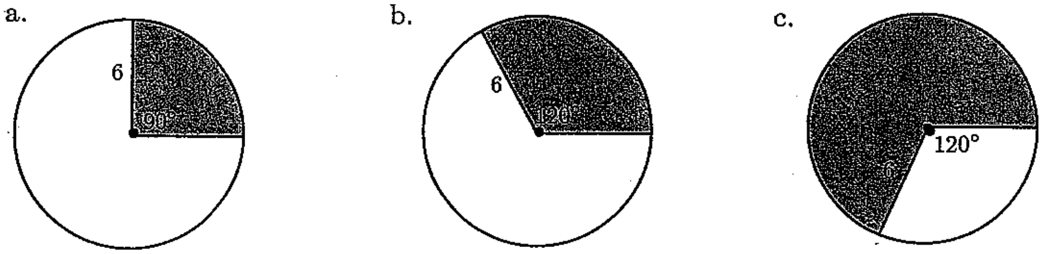
14. Pictured is a circle of diameter 9 inscribed in a square. There is also an octagon $ABCDEFGH$ which is **not** regular.
   a. Do the circle and the octagon appear to have about the same area?
   b. What is the area of the octagon?
   c. How does this result compare with the Egyptian formula for the area of a circle?



Exercises 8.6

1. Find the area of the shaded region in each of the following.

a. b. c.



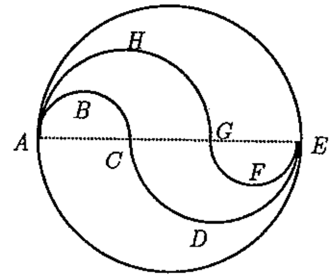
2. What is the area of a 30° sector in a circle of radius 3?

3. a. In a circle of radius 4, how large a central angle is needed so that one of the two sectors created has an area exactly double the other?

   b. Does the radius of the circle matter in this question?

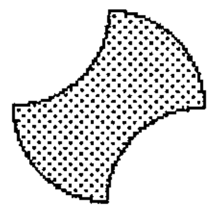
4. When a chord cuts off a 90° arc in a circle, two segments of a circle are formed. Show that the area of the larger one is more than 10 times that of the smaller one.

5. In the figure, the large circle has radius 6. Diameter $\overline{AE}$ is trisected at C and G, and then four semicircles are drawn.
   a. What is the area of the region $ABCDEFGH$?
   b. What is the perimeter of region $ABCDEFGH$?

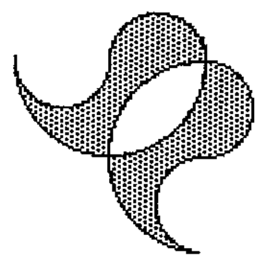


6. a. What is the area of a 60° segment in a circle of radius 6?
   b. What is the area of a 60° segment in a circle of radius r?

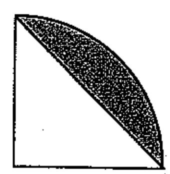
7. In the figure, all four arcs are congruent 90° arcs. The radius of each arc is 2. What is the area of the region?



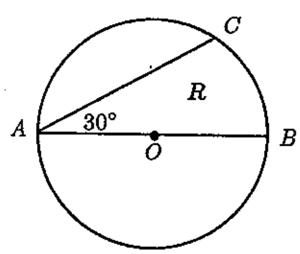
8. In the figure, the large arcs are semicircles of radius 2 and the small arcs are semicircles of radius 1. The centers and endpoints of the large semicircles are endpoints of the small semicircles. What is the area of the shaded region?



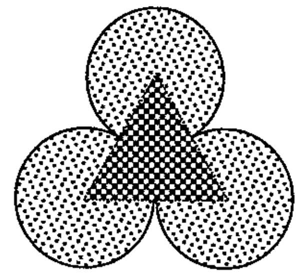
9. Pictured is a 90° segment of a circle of radius 4. What is the area of the segment?



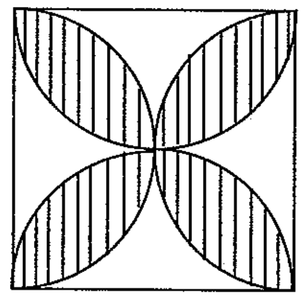
10. In the figure, R is the region bounded by chords $\overline{AB}$ and $\overline{AC}$ along with arc $\overset{\frown}{BC}$. $m\angle A = 30$ and the radius of the circle is 6.
   a. What is the area of R?
   b. What is the perimeter of R?



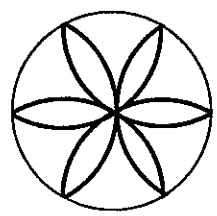
11. A trefoil is formed by the arcs of three congruent circles that are mutually tangent to each other. The centers of the circles are at the vertices of an equilateral triangle of side 6. Find the perimeter and the area of the trefoil.



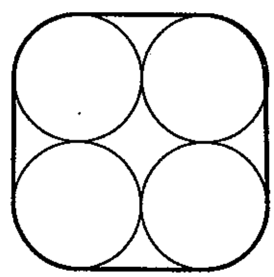
12. A quatrefoil is formed by arcs of four congruent circles using sides of a square as diameters. The quatrefoil pictured to the right is in a square of side 4.
    a. What is the perimeter of the quatrefoil?
    b. What is the area of the quatrefoil?



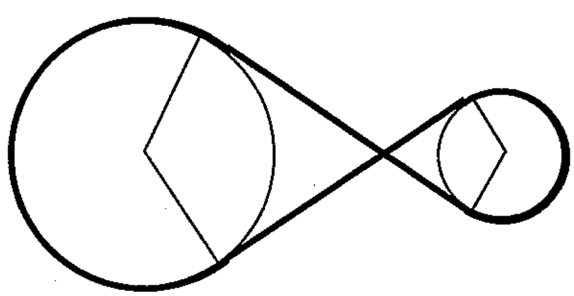
13. In the figure to the right, the outside circle has a radius of 6. Each of the dark arcs also has radius 6. What is the area enclosed by the six petals of the pattern?



14. Four congruent circles, each with a radius of 2, have a band that goes around them tightly, forcing the circles to be tangent to one another.
    a. How long is the band?
    b. What is the total area enclosed?

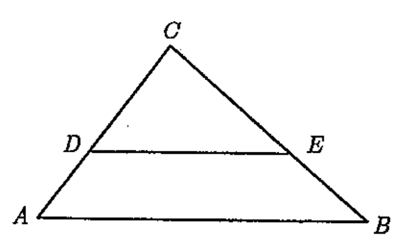


15. In the figure to the right, a belt goes around two wheels (when one wheel turns one direction, the belt makes the other go in the opposite direction). The radii of the circles are 8 inches and 4 inches, and the centers are 20 inches apart. How long is the belt (to the nearest 0.01 inch)?



Exercises 8.7

1. In the figure, $\overline{DE} \parallel \overline{AB}$.



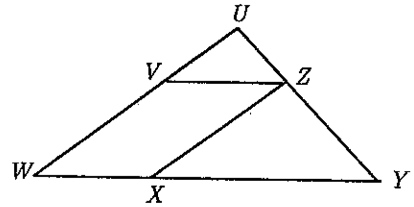
   a. If $AB = 8$ and $DE = 5$, what is $\frac{a(\Delta ABC)}{a(\Delta DEC)}$?

   b. If $AD = 2$ and $DC = 3$, what is $\frac{a(\Delta ABC)}{a(\Delta DEC)}$?

   c. If $AB = 7$ and $DE = 4$, what is $\frac{a(\Delta DEC)}{a(ABED)}$?

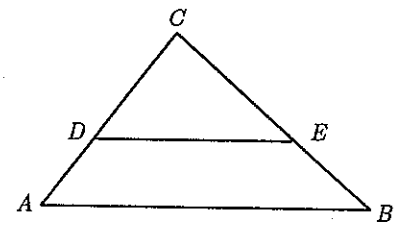
   d. If $BE = 7$ and $EC = 10$, what is $\frac{a(\Delta DEC)}{a(ABED)}$?

2. In the figure, $\overline{VZ} \parallel \overline{WY}$ and $\overline{XZ} \parallel \overline{WU}$. If $WY = 7$ and $VZ = 3$, what is $\frac{a(\Delta UVZ)}{a(\Delta XYZ)}$?



3. The sides of a triangle are 6, 8, and 10. What are the sides of a similar triangle with four times the area? Twice the area?

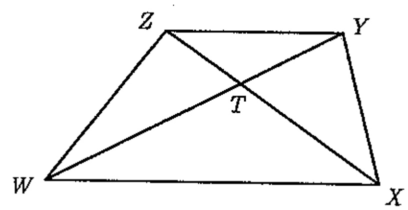
4. In the figure, $\overline{DE} \parallel \overline{AB}$. If $AD = 4$ and the area of ΔDEC equals the area of trapezoid $ABED$, then what is CD?



5. Given trapezoid $WXYZ$ with $\overline{WX} \parallel \overline{YZ}$. $WX = 8$ and $YZ = 5$. Determine each of the following ratios.

a. $\frac{a(\Delta WXT)}{a(\Delta YZT)}$ b. $\frac{a(\Delta WZY)}{a(\Delta WXY)}$

c. $\frac{a(\Delta WYZ)}{a(\Delta XYZ)}$ d. $\frac{a(\Delta WZT)}{a(\Delta XYT)}$



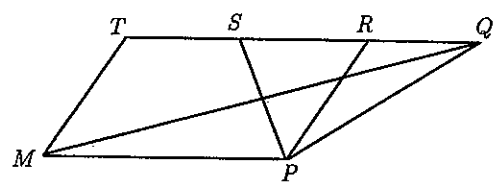
6. In ΔADE, points B and C are on $\overline{AD}$ with $AB = BC = CD$. Prove that $a(\Delta BDE) = 2 \cdot a(\Delta ABE)$.

7. In ΔABC, D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AD}$, and F is the midpoint of $\overline{BE}$. Prove that $a(\Delta EFD) = \frac{1}{8} \cdot a(\Delta ABC)$.

8. In the figure, $MPRT$ is a parallelogram. $TS = SR = RQ$. Determine the following ratios.

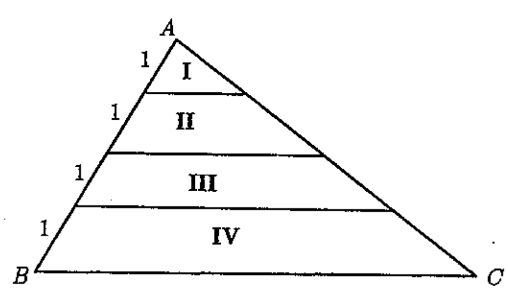
a. $\frac{a(\Delta PRS)}{a(\Delta PRQ)}$ b. $\frac{a(\Delta PMQ)}{a(\Delta PQS)}$

c. $\frac{a(\Delta PMQ)}{a(MPRT)}$ d. $\frac{a(\Delta PQR)}{a(MPST)}$



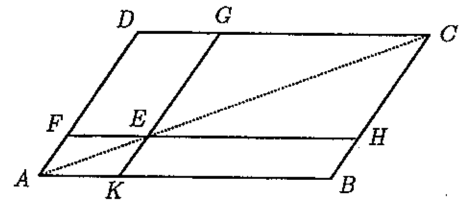
9. ΔABC and ΔDEF are equilateral triangles. An altitude of ΔDEF is the same length as a side of ΔABC. What is the ratio of the areas of the two triangles?

10. In ΔABC, lines are drawn parallel to $\overline{BC}$ cutting $\overline{AB}$ into four segments of length 1. Determine the areas of the four resulting regions if the area of ΔABC is 144.

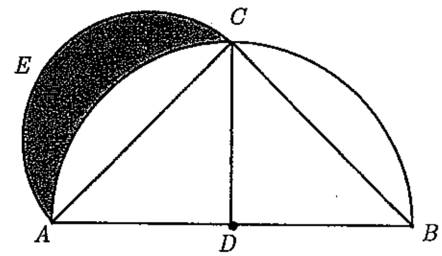


11. Medians $\overline{AD}$ and $\overline{CE}$ of ΔABC intersect at F. Prove that $a(\Delta AEF) = a(\Delta CDF)$.

12. $ABCD$ is a parallelogram. E is on diagonal $\overline{AC}$. $\overline{FEH} \parallel \overline{AB}$ and $\overline{GEK} \parallel \overline{AD}$. Prove that $\frac{a(KBHE)}{a(DFEG)} = 1$.



13. This proof is due to the Greek geometer Hippocrates of Chios. In the figure to the right, ΔABC is an isosceles right triangle inscribed in a semicircle with its center at D. $\overset{\frown}{AEC}$ is a semicircle drawn on diameter $\overline{AC}$.



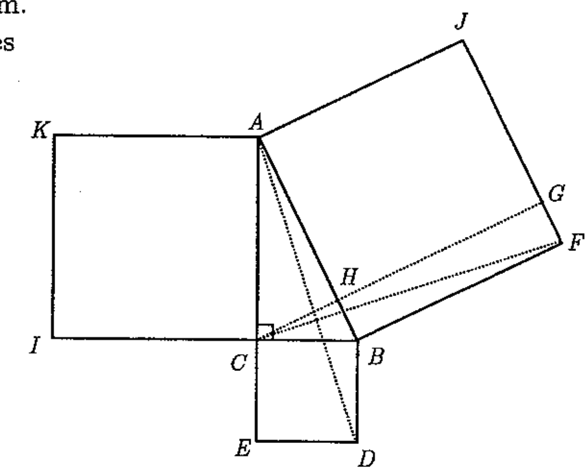
a. What is the ratio of the diameters of semicircles $\overset{\frown}{ACB}$ and $\overset{\frown}{AEC}$?

b. What is the ratio of the areas of semicircle $\overset{\frown}{ACB}$ and $\overset{\frown}{AEC}$?

c. What is the ratio of the areas of sector $\triangledown ADC$ and semicircle $\overset{\frown}{AEC}$?

d. How does the area of the shaded region (a "lune") compare to the area of ΔADC?

14. This is Euclid's proof of the Pythagorean Theorem. In the figure, squares are drawn on the three sides of right ΔABC. Then $\overline{CG}$ is drawn parallel to $\overline{BF}$, and $\overline{AD}$ and $\overline{CF}$ are drawn.



a. Show that ΔDBA congruent to ΔCBF.

b. Show that $a(\Delta DBA) = \frac{1}{2} \cdot a(DBCE)$.

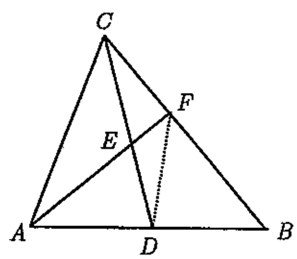
c. Show that $a(\Delta CBF) = \frac{1}{2} \cdot a(BFGH)$.

d. Show that $a(DBCE) = a(BFGH)$.

e. Now draw $\overline{BK}$ and $\overline{CJ}$. Show that $a(ACIK) = a(AJGH)$.

15. Given: D is the midpoint of $\overline{AB}$.
E is the midpoint of $\overline{CD}$.

Prove: $a(\Delta AFD) = a(\Delta BFD) = a(\Delta AFC)$

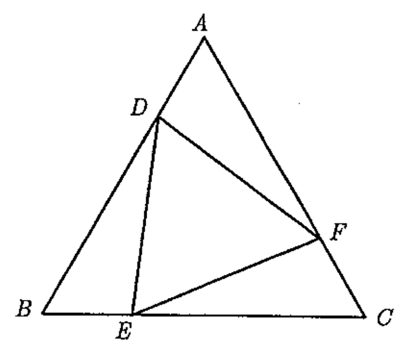


16. In the figure, ΔABC is equilateral. D, E, and F are on $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively, so that $AD = BE = CF$. Determine $\frac{a(\Delta DEF)}{a(\Delta ABC)}$ in the following cases.

a. $AD = DB$

b. $AD = \frac{1}{4}(AB)$

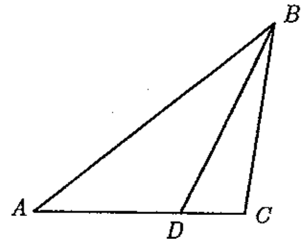
c. $AD = x$ and $EC = y$



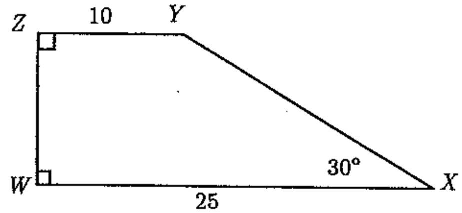
Chapter 8 Review Exercises

1. Find the area of a rhombus with sides of 17 and a longer diagonal of length 30.

2. The area of ΔABC is 56. $AC = 14$ and $DC = 4$. Find the areas of ΔABD and ΔCBD.



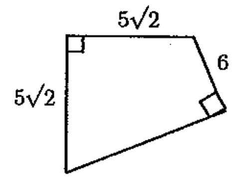
3. Find the area of quadrilateral $WXYZ$.



4. An equilateral triangle is inscribed in a circle of radius 4. Find the area that is inside the circle but outside the triangle.

5. Find the area of a regular hexagon whose perimeter is 18.

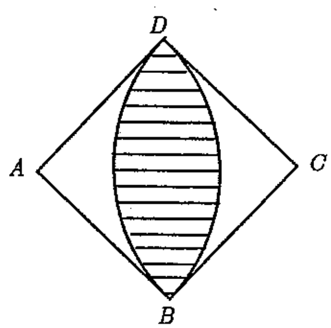
6. Find the area of the quadrilateral shown.



7. Identify each of the following as **True** or **False**.
   a. The circumference of a circle with diameter 10 is 10π.
   b. If two circles have the same circumference, then they must be congruent.
   c. $\pi \neq 3.14$
   d. The formula for the area of a circle with diameter d is $\frac{1}{2}\pi d^2$.

8. In a circle of area 81π, a sector with an area of 45π has an arc of what degree measure?

9. In the figure to the right, $ABCD$ is a square of side s, and quarter-circles centered at A and C are drawn. Determine the area of the region that is inside the square but outside the leaf-shaped piece in the middle (the **un**-shaded part of the figure).

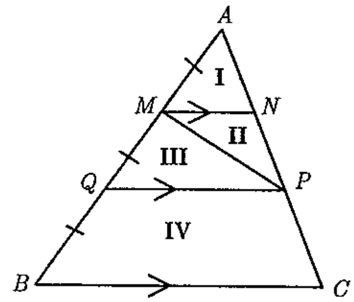


10. Find the area of a circle inscribed in a 6–8–10 right triangle.

11. What is the area of a 30° segment in a circle of radius 6?

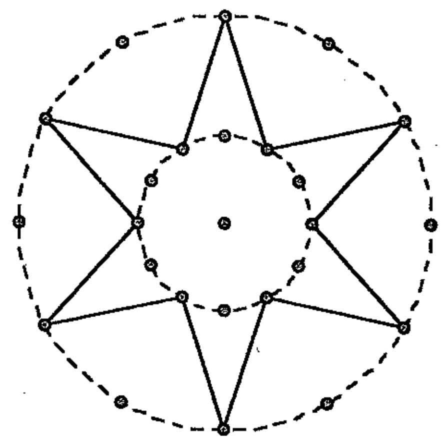
12. In quadrilateral $ABCD$, the diagonals intersect at E. Diagonal $\overline{AC}$ is extended through C to a point F so that $CF = AE$. Prove that $a(\Delta BDF) = a(ABCD)$.

13. In the figure to the right, $a(MNPQ) = 48$. Determine the areas of the four regions into which the triangle has been subdivided.



14. Determine the area of trapezoid $ABCD$ if $\overline{AB} \parallel \overline{CD}$, $m\angle A = 45$, $m\angle B = 60$, $AD = 6\sqrt{6}$, and $AB = 12 + 6\sqrt{3}$.

15. A six-pointed star is formed in the following way. Take two concentric clocks, a large one of radius R and a small one of radius r. Join the point at 12 on the large clock to the point at 1 on the small clock, to the 2 on the large clock, to the 3 on the small clock, to the 4 on the large clock, and so on, ending up by joining the 11 on the small clock to the 12 on the large clock. What is the area of the star?



16. Prove Pappus' extension of the Pythagorean Theorem. Let ΔABC be given, with parallelograms $ABED$ and $BCFG$ drawn on two of the sides. Let $\overline{DE}$ and $\overline{FG}$ be extended to intersect at H. If $\overline{AY}$ and $\overline{CX}$ are congruent to $\overline{HB}$ and also parallel to $\overline{HB}$, then $a(AYXC) = a(ABED) + a(BCFG)$.