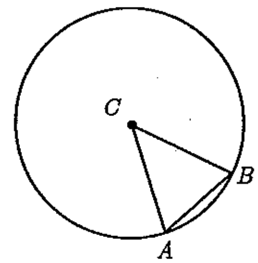
Exercises 7.1

1. Identify each of the following statements as **TRUE** or **FALSE**.
   a. A diameter of a circle is the longest chord of the circle.
   b. Any triangle formed by a chord and the radii drawn to the endpoints of the chord is equilateral.
   c. The center of the circle circumscribed about a right triangle is the midpoint of the hypotenuse of the triangle.
   d. The center of the circle circumscribed about an obtuse triangle lies in the exterior of the triangle.
   e. Every quadrilateral has a circumscribed circle.
   f. A parallelogram that is not a rectangle can never be inscribed in a circle.

2. In $\odot C$ shown to the right, $m\angle ACB = 40$. What is $m\angle A$?



3. Draw six congruent circles that all pass through a common point. What can you say about the centers of those circles?

4. How would you locate the center of a circle when only a portion of the circle is given?

5. Prove the Common Chord Theorem: If two different circles intersect in two points, then the line joining the centers of the circles is perpendicular to the common chord (the segment that is a chord of both circles at the same time).

6. Two circles of radii 13 and 15 intersect in two points and the length of their common chord is 24. Find the two possible distances between the centers of the circles.

7. The centers of two circles with radii 10 and 17 are 21 units apart. Find the length of the common chord.

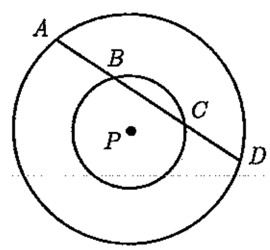
8. Prove **Theorem 7.2**: A radius that bisects a chord of a circle is perpendicular to that chord.

9. Prove **Theorem 7.3**: A radius that is perpendicular to a chord of a circle bisects that chord.

10. Prove **Theorem 7.4**: The perpendicular bisector of a chord of a circle passes through the center of the circle.

11. Chord $\overline{AB}$ is drawn in $\odot C$. Use Theorem 7.4 to solve the following.
    a. How far is $\overline{AB}$ is from the center of the circle if $AB = 12$ and $AC = 10$?
    b. What is the radius of the circle if $AB = 6$ and $\overline{AB}$ is 5 units from the center?

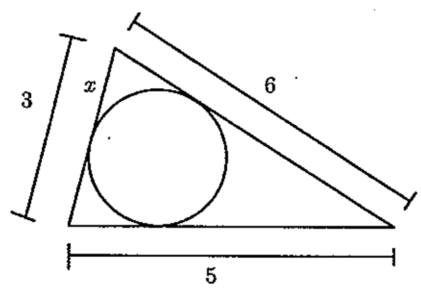
12. Given concentric circles with common center P. Chord $\overline{AD}$ of the larger circle intersects the smaller circle at B and C. Prove that $\overline{AB} \cong \overline{CD}$.



13. The radius of a circle is 17. Point P is on a chord of length 30, and is 9 units from one end of the chord. How far is P from the center of the circle?

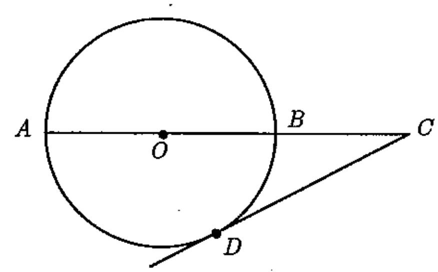
Exercises 7.2

1. In the figure, the circle is inscribed in the triangle whose sides are 3, 5, and 6. Determine x.



2. A circle is inscribed in a triangle whose sides are 7, 8, and 9. Determine the length of the six segments from the vertices to the points of tangency.

3. $\overline{AOB}$ is a diameter of $\odot O$. $\overline{CD}$ is tangent to the circle at D. $CD = 6$ and $BC = 4$. Determine the radius of the circle.



4. A circle is inscribed in an equilateral triangle of side 12. What is the radius of the circle?

5. Point W is 8 units from the center of a circle of radius 3. What is the length of a tangent segment drawn to the circle from W?

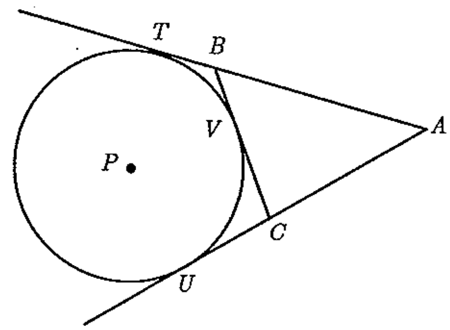
6. a. Concentric circles have radii 6 and 10. A chord of the larger circle is tangent to the small one. How long is the chord?

   b. The larger of two concentric circles has a radius of 12. A chord of that circle has length 8 and is tangent to the smaller circle. What is the radius of the smaller circle?

7. In circle O, radius $\overline{OC}$ is extended its own length to B. From B, tangent $\overline{BA}$ is drawn. Prove that ΔOAC is equilateral.

8. Given: $\overrightarrow{AT}$ and $\overrightarrow{AU}$ are tangent to $\odot P$ at T and U, respectively. $\overline{BC}$ is tangent to $\odot P$ at V.

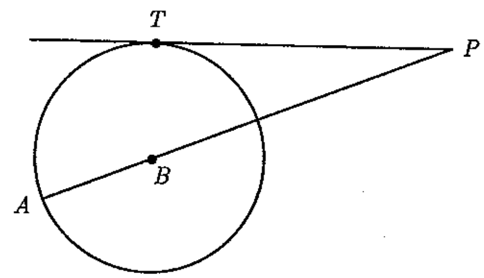
Prove: The perimeter of $\Delta ABC = 2\cdot(AT)$



9. a. Isosceles trapezoid $WXYZ$ is circumscribed about a circle. Prove that the length of the median of the trapezoid is one-fourth of the trapezoid's perimeter.

   b. Is the result of part (a) true for any trapezoid circumscribed about a circle?

10. In the figure, $\overrightarrow{PT}$ is tangent to $\odot B$ at T. $PT = 4k$ and $PA = 8k$. Determine the radius of the circle in terms of k.

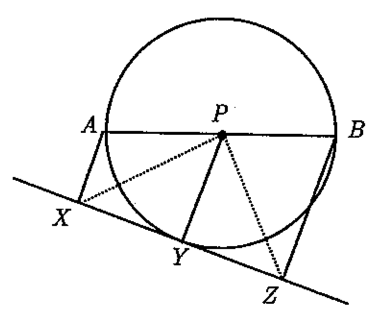


11. Determine the radius of a circle inscribed in a triangle with sides of length 5, 5, and 6.

12. Two tangents are drawn to a circle of radius r from a point that is r units from the circle. How far from the center of the circle is the chord that joins the points of tangency?

13. Given: $\overline{AB}$ is a diameter of $\odot P$. $\overleftrightarrow{XY}$ is tangent to the circle at Y. $\overline{AX}$ and $\overline{BZ}$ are perpendicular to $\overleftrightarrow{XY}$.

Prove: $PX = PZ$



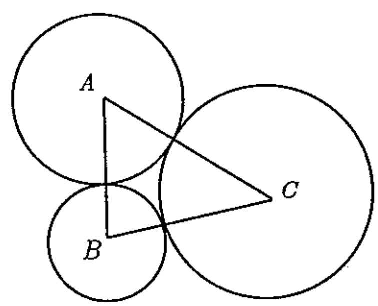
14. Quadrilateral $MNOP$ is circumscribed about a circle. Prove that $MN + OP = MP + ON$.

15. A right triangle has legs a and b and hypotenuse c. What is the radius of the inscribed circle (in terms of a, b, and c)?

16. Line ℓ is tangent to circle P. From the endpoints of diameter $\overline{AB}$, segments $\overline{AX}$ and $\overline{BY}$ are drawn perpendicular to line ℓ. Prove that $AX + BY = AB$.

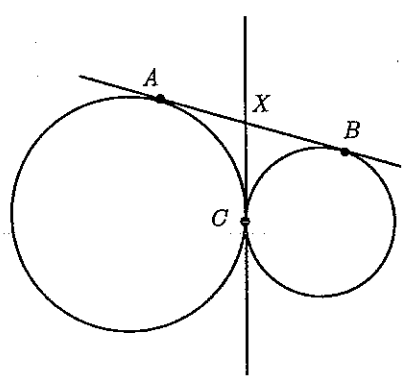
Exercises 7.3

1. Circles A, B, and C are externally tangent to each other. $AB = 15$, $BC = 21$, and $AC = 26$.
   a. What are the radii of the three circles?
   b. Why is Theorem 7.8 important in this problem?



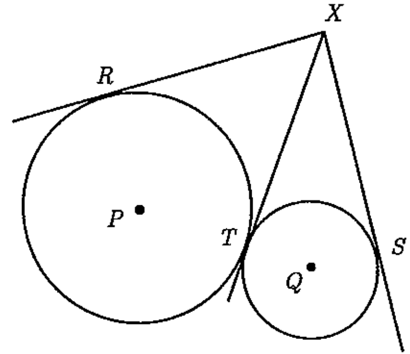
2. <u>Given</u>: $\overleftrightarrow{AB}$ is externally tangent to the two circles at A and B. $\overleftrightarrow{CX}$ is internally tangent to the two circles at C.

   <u>Prove</u>: X is the midpoint of $\overline{AB}$.



3. Given: $\odot P$ and $\odot Q$ are externally tangent to each other at T. $\overrightarrow{XR}$ is tangent to $\odot P$ at R. $\overrightarrow{XS}$ is tangent to $\odot Q$ at S.

Prove: $\overline{XR} \cong \overline{XS}$

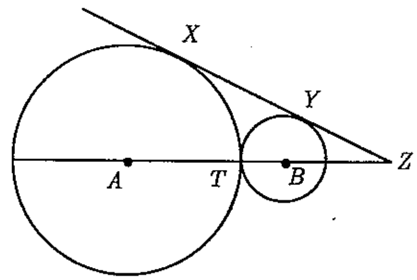


4. Circles A and B are tangent to each other at T. The common external tangent $\overleftrightarrow{XY}$ intersects the line of centers at Z. Determine XY and YZ in each of the following cases.

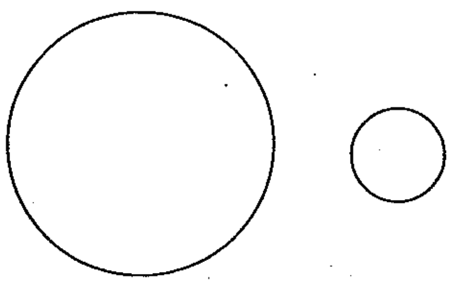
a. $AT = 8$; $TB = 2$

b. $AT = 6$; $TB = 2$

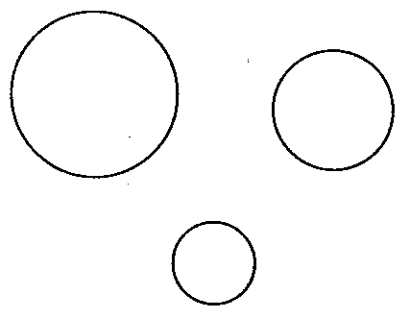
c. $AT = 2$; $TB = 1$



5. a. How many circles are there that are tangent to both of the circles pictured?



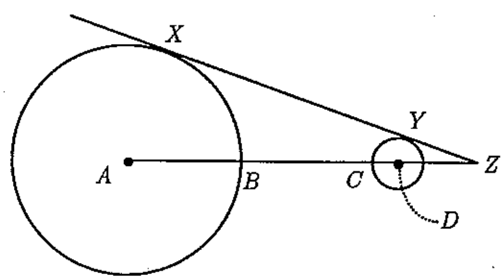
b. How many circles are there that are tangent to all three of the circles pictured?



6. $\overleftrightarrow{XY}$ is a common external tangent. Determine XY and YZ in each of the following cases.

a. $AB = 6$; $BC = 6$; $CD = 1$

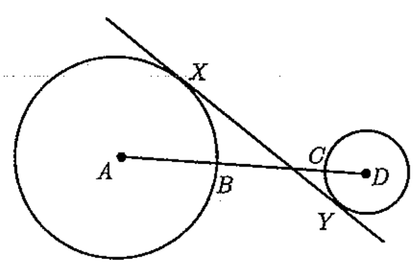
b. $AB = 4$; $BC = 3$; $CD = 2$



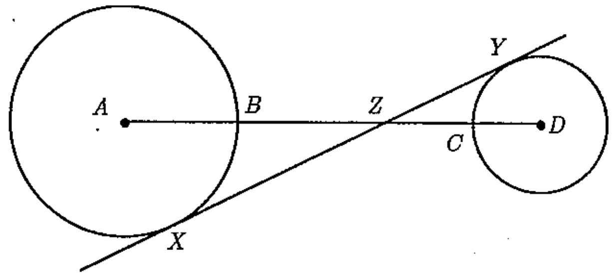
7. $\overleftrightarrow{XY}$ is a common internal tangent to $\odot A$ and $\odot D$. Determine XY in each of the following cases.

a. $AB = 6$; $BC = 6$; $CD = 3$

b. $AB = 8$; $BC = 2$; $CD = 2$



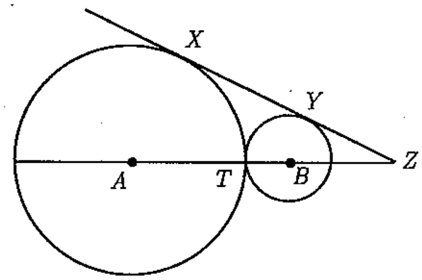
8. $\odot A$ and $\odot D$ have radii 6 and 2, respectively. $\overleftrightarrow{XY}$ is a common internal tangent. If $BC = 9$, determine XY and YZ.



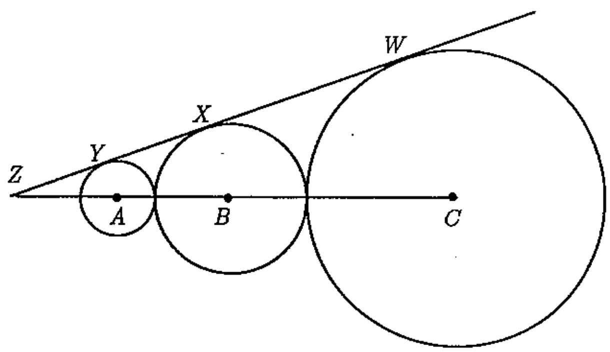
9. Circles of radii 4 and 12 have their centers 17 units apart. Find the lengths of the common external and internal tangent segments.

10. Circles of radii 5 and 12 have their centers 25 units apart. Find the lengths of the common external and internal tangent segments.

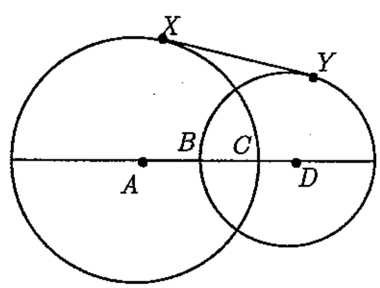
11. $\odot A$ and $\odot B$ are externally tangent at T. $\overleftrightarrow{XY}$ is a common external tangent to the two circles, and $\angle Z$ is a 30° angle. What is the ratio of the radii of the two circles?



12. $\odot A$, $\odot B$, and $\odot C$ are externally tangent as shown, with A, B, and C collinear. $\overleftrightarrow{XY}$ is a common external tangent to all three circles. The radius of $\odot A$ is 1 and the radius of $\odot B$ is 2.
   a. What is the radius of $\odot C$?
   b. What is the distance AZ?



13. Find the length of the common tangent segment $\overline{XY}$ in each of the following cases.
   a. $AB = 6$; $BC = 3$; $CD = 1$
   b. $AB = 10$; $BC = 4$; $CD = 1$



14. Two circles are tangent externally and each is tangent internally to a third circle. The segments joining the centers have lengths 15, 11, and 6.
   a. Draw a reasonable figure, trying to make each point of tangency collinear with the appropriate circle centers.
   b. Determine the radius of each circle.

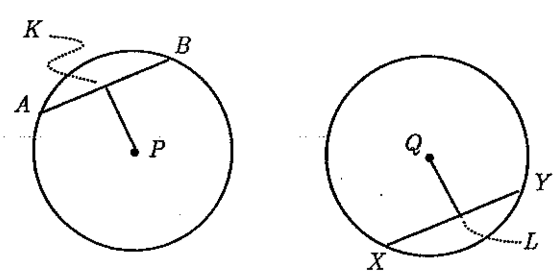
Exercises 7.4

1. Consider a circle with $\widehat{AB}$ have measure 100. Point P is the midpoint of that $\widehat{AB}$. If $m\widehat{PBC} = 160$, find the measure of the minor arc $\widehat{AC}$.

2. Prove **Theorem 7.10:** In the same or congruent circles, congruent chords have congruent arcs.

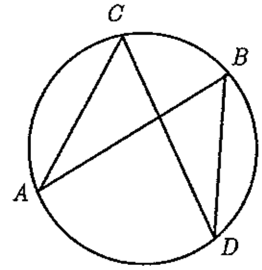
3. <u>Given</u>: $\odot P \cong \odot Q$; $\overline{AB} \cong \overline{XY}$; $\overline{PK} \perp \overline{AB}$; $\overline{QL} \perp \overline{XY}$.

   <u>Prove</u>: $\overline{PK} \cong \overline{QL}$

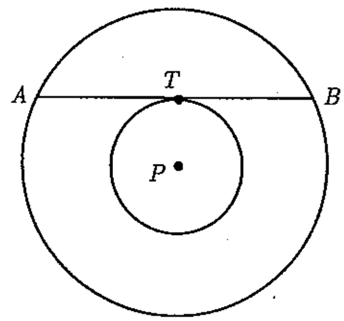


4. Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$



5. Chord $\overline{AB}$ of the larger of two concentric circles is tangent to the smaller circle at T. Prove that T is the midpoint of $\overline{AB}$.



6. Point P is in the interior of a circle of radius 13, and is 1 unit from the circle. How long is the chord through P that is perpendicular to the diameter through P?

7. Point P is inside $\odot O$. How do you get a chord of $\odot O$ to pass through P, for which P is the midpoint?

8. Given: In $\odot P$, $m\overset{\frown}{AB} = 120$, $m\overset{\frown}{BC} = 80$, D is the midpoint of $\overset{\frown}{AC}$.

Depending on how the figure is drawn and how $\overset{\frown}{AC}$ is interpreted, there are **four** different results you can get for minor $m\overset{\frown}{AD}$. What are they?

9. A square is inscribed in a circle of radius 8. How far are the sides of the square from the center of the circle?

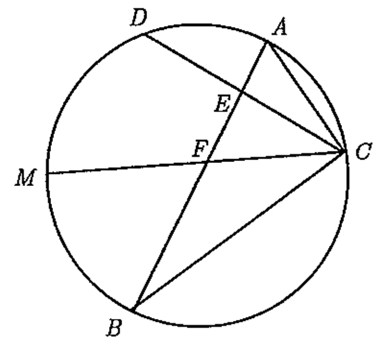
10. Two concentric circles are drawn. Chord $\overline{AB}$ of the larger circle intersects the smaller circle at points C and D. Prove that $\overline{AC} \cong \overline{BD}$.

11. A regular hexagon is inscribed in a circle of radius r. What is the perimeter of the hexagon?

12. A regular hexagon is circumscribed about a circle of radius r. What is the perimeter of the hexagon?

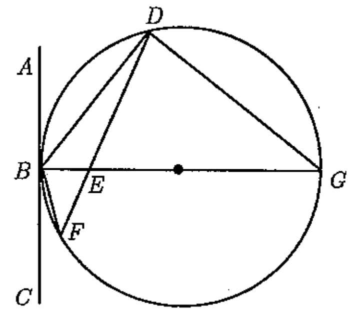
Exercises 7.5

1. In the figure, $\overline{AB}$ is a diameter, $m\overset{\frown}{AC} = 50$, $m\overset{\frown}{BD} = 5 \cdot m\overset{\frown}{AD}$, and M is the midpoint of $\overset{\frown}{BD}$. Determine each of the following.



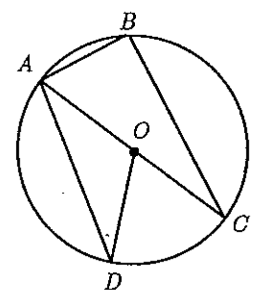
   a. $m\angle ACB$ b. $m\angle ABC$
   c. $m\overset{\frown}{AD}$ d. $m\angle ACD$
   e. $m\overset{\frown}{DM}$ f. $m\angle MCB$
   g. $m\angle AEC$ h. $m\angle MFB$

2. In the figure, $\overline{BG}$ is a diameter of the circle, $\overleftrightarrow{ABC}$ is tangent to the circle at B, $m\overset{\frown}{BD} = 80$, and $m\angle BEF = 60$. Determine each of the following.



   a. $m\angle BDG$ b. $m\angle F$
   c. $m\overset{\frown}{DG}$ d. $m\angle DBG$
   e. $m\overset{\frown}{BF}$ f. $m\angle BDF$
   g. $m\angle GBF$ h. $m\angle CBF$

3. In the figure to the right, $\overline{AC}$ is a diameter of $\odot O$. $m\overset{\frown}{BC} = 130$ and $m\overset{\frown}{DC} = 60$. Find:



   a. $m\angle B$ b. $m\angle C$
   c. $m\angle D$ d. $m\overset{\frown}{AD}$

4. Prove each of the corollaries to Theorem 7.11. (Each proof should take only a couple of sentences, since these are corollaries.)
   a. An angle inscribed in a semicircle is a right angle.
   b. Two inscribed angles that intercept the same arc are congruent.
   c. The opposite angles of an inscribed quadrilateral are supplementary.

5. $\overline{AB}$ and $\overline{CD}$ are two chords of a circle intersecting at X. If $m\angle XAC = 32$ and $m\angle XCA = 40$, find the measures of $\angle XBD$ and $\angle XDB$.

6. Prove: A parallelogram inscribed in a circle is a rectangle.

7. ΔXYZ is inscribed in a circle with $m\angle X = 20$, $m\angle Y = 50$, and $m\angle Z = 110$. The bisectors of the angles of the triangle are extended to intersect the circle at A, B, and C. What are the measures of the angles of ΔABC?

8. ΔXYZ is inscribed in a circle. The measures of the angles of this triangle are $2x$, $2y$, and $2z$. The bisectors of the angles of ΔXYZ are extended to meet the circle at points A, B, and C. Determine the measures of the angles of ΔABC.

In problems 9 and 10, $ABCD$ is a square inscribed in a circle and point P lies on $\overset{\frown}{CD}$.

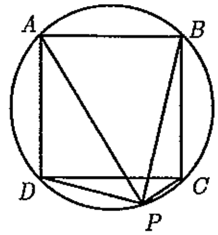


Figure for problems 9 and 10

9. Prove that $\overline{PA}$ and $\overline{PB}$ trisect $\angle DPC$.

10. If the circle has radius r, determine PA in terms of PB and PD,

11. Octagon $ABCDEFGH$ is inscribed in a circle. Arcs $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, ... , $\overset{\frown}{HA}$ have measures 10, 20, 30, ... , 80 **in order around the circle.**
   a. Which two vertices are the endpoints of a diameter of the circle?
   b. Which angle is the largest angle of the octagon? What is its measure?
   c. Explain how you can tell that $\overline{DH} \perp \overline{BF}$.

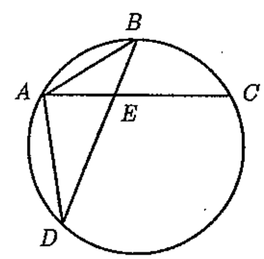
12. Quadrilateral $ABCD$ is inscribed in a circle with $\overline{AB} \cong \overline{BC} \cong \overline{CD}$. Prove that $\overline{AD} \parallel \overline{BC}$.

13. Prove indirectly that if the opposite angles of a quadrilateral are supplementary, then there is a circle that can be circumscribed about the quadrilateral. (Start with the circle that passes through three of the vertices and assume that it does **not** pass through the fourth vertex.)

14. <u>Given</u>: B is the midpoint of $\overset{\frown}{AC}$.

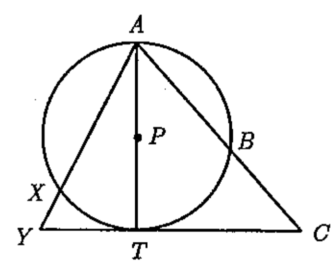
    <u>Prove</u>: a. $\Delta ABE \sim \Delta DBA$

    b. $AB^2 = BD \cdot BE$



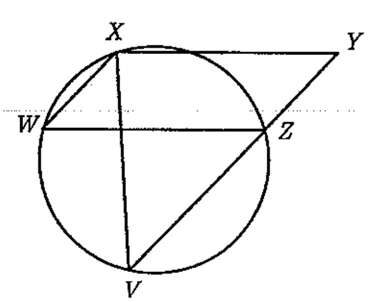
15. $\overline{AB}$ is a diameter of $\odot P$. A tangent to the circle is drawn at A. From point C on the tangent, $\overline{CB}$ is drawn, intersecting the circle at D. Prove that $AD^2 = BD \cdot CD$.

16. $\overline{APT}$ is a diameter of $\odot P$ and $\overline{YTC}$ is tangent to the circle at T. Prove that $AX \cdot AY = AB \cdot AC$.

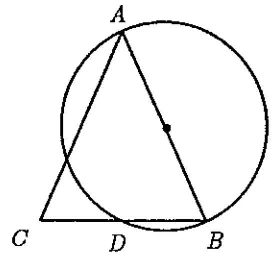


17. <u>Given</u>: $WXYZ$ is a parallelogram with points W, X, and Z lying on a circle. $\overline{YZ}$ is extended to intersect the circle again at V.

    <u>Prove</u>: ΔYXV is isosceles.

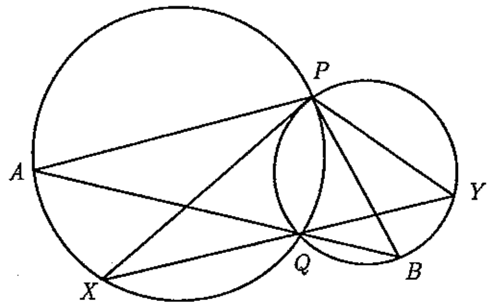


18. ΔABC is isosceles with $\overline{AB} \cong \overline{AC}$. $\overline{AB}$ is a diameter of the circle. Prove that D is the midpoint of $\overline{BC}$.

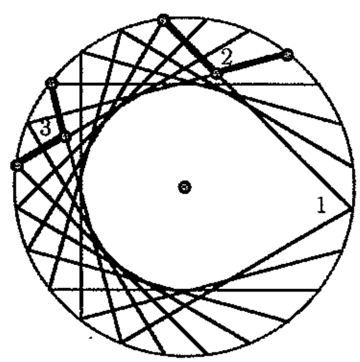


19. Given: Two circles intersect at P and Q. Secants $\overline{AB}$ and $\overline{XY}$ both pass through Q.

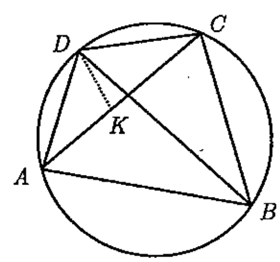
Prove: $\angle APB \cong \angle XPY$



20. In the figure to the right, there are 24 equally spaced points around the circle. Determine:
   a. $m\angle 1$
   b. $m\angle 2$
   c. $m\angle 3$



21. Prove **Theorem 7.13 (Ptolemy's Theorem):** If $ABCD$ is a quadrilateral inscribed in a circle, then $AC \cdot BD = AB \cdot CD + BC \cdot AD$.
   a. Draw $\overline{DK}$ so that $\angle ADK \cong \angle BDC$. Then prove $AB \cdot CD = CK \cdot BD$.
   b. Next prove $BC \cdot AD = AK \cdot BD$.
   c. Add and simplify your results.

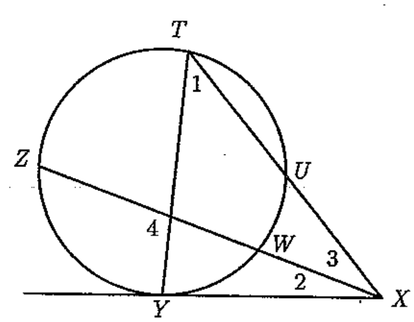


22. Use Ptolemy's Theorem to establish the following result. If ΔABC is an equilateral triangle inscribed in a circle, and if P is any point on $\overset{\frown}{BC}$, then $PA = PB + PC$.

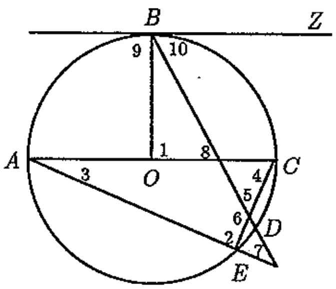
23. Let ΔXYZ be a right triangle inscribed in a circle, with the right angle at Y. Let the diameter of the circle be c, and let the legs of the right triangle have lengths a and b. Draw diameter $\overline{YW}$ and complete quadrilateral $WXYZ$. What does Ptolemy's Theorem say about $WXYZ$?

Exercises 7.6

1. In the figure to the right, $\overleftrightarrow{XY}$ is tangent to the circle at Y.
   a. Identify the arc(s) that are intercepted by $\angle 1$.
   b. Identify the arc(s) that are intercepted by $\angle 2$.
   c. Identify the arc(s) that are intercepted by $\angle 3$.



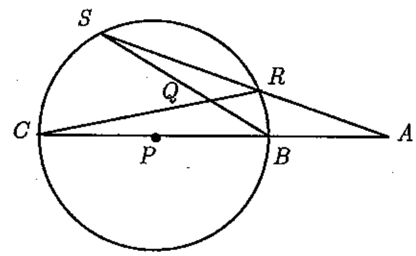
2. In the figure, $\overleftrightarrow{BZ}$ is tangent to $\odot O$ $\overline{AC}$ is a diameter. $m\overset{\frown}{BC} = 90$, $m\overset{\frown}{CD} = 30$, and $m\overset{\frown}{DE} = 20$. Determine the measure of each of the angles numbered 1 through 10.



3. In the figure, $m\angle A = 25$ and $m\angle ABS = 140$. Determine each of the following.

a. $m\overset{\frown}{RB}$ b. $m\overset{\frown}{SC}$

c. $m\angle CQB$ d. $m\angle CRS$



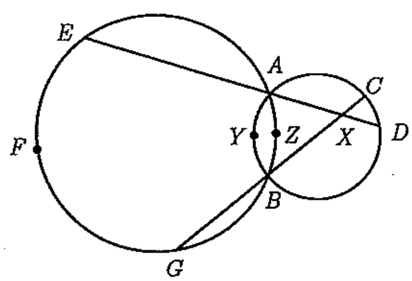
4. Two tangents drawn to a circle from an external point meet at a 36° angle. Find the measures of the two arcs intercepted by the tangents.

5. Prove: The diagonals of a trapezoid inscribed in a circle are congruent.

6. Trapezoid $ABCD$ is inscribed in a circle. The bases $\overline{AB}$ and $\overline{CD}$ are on opposite sides of the center. $m\overset{\frown}{AB} = 56$ and $m\overset{\frown}{CD} = 114$. Find the measures of the angles of the trapezoid and the measure of the angle formed by extending $\overline{AD}$ and $\overline{BC}$ until they meet.

7. Radii $\overline{PA}$ and $\overline{PB}$ of $\odot P$ meet at a 36° angle. Diameter $\overline{DC}$ is drawn parallel to $\overline{AB}$. Find the measures of the angles formed by the diagonals of quadrilateral $ABCD$.

8. Two circles intersect at A and B.
$m\angle AXB = 70$, $m\overset{\frown}{CD} = 20$, and $m\overset{\frown}{EFG} = 160$.
Find $m\overset{\frown}{AYB} - m\overset{\frown}{AZB}$.



9. Prove the second case of **Theorem 7.18**: The measure of an angle formed by a secant and a tangent to a circle from an external point is half the difference of the intercepted arcs.

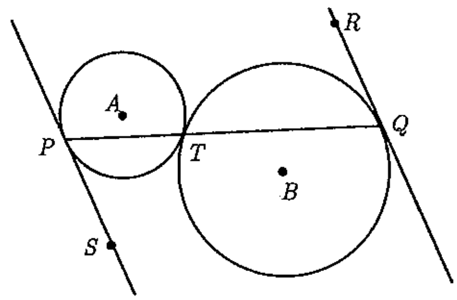
10. Prove the third case of **Theorem 7.18**: The measure of an angle formed two tangents to a circle from an external point is half the difference of the intercepted arcs.

11. Given: $\overline{AB}$ and $\overline{AC}$ are congruent chords in a circle.
Line ℓ is tangent to the circle at point C.

Prove: A is equidistant from line ℓ and chord $\overline{BC}$.

12. Given: $\odot A$ and $\odot B$ are tangent to each other at T. $\overline{PQ}$ passes through T. $\overleftrightarrow{PS}$ and $\overleftrightarrow{QR}$ are tangents.

Prove: $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$



13. What is the measure of the angle made by the hands of a clock at the following times?

a. 1:00   b. 4:00   c. 6:30

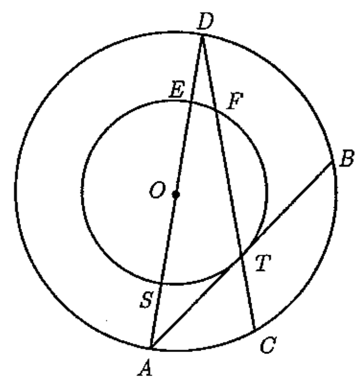
d. 2:20   e. 3:40   f. 9:10

14. In the figure, the two circles are concentric. Chord $\overline{AB}$ of the larger circle is tangent to the smaller circle at T. $m\overset{\frown}{DB} = 76$ and $m\overset{\frown}{AC} = 38$. Determine each of the following.

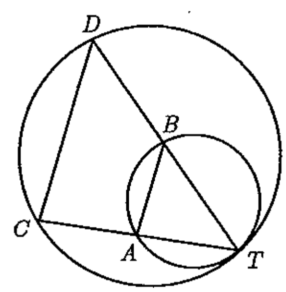
a. $m\angle A$ b. $m\angle D$

c. $m\overset{\frown}{ST}$ d. $m\overset{\frown}{EF}$

e. $m\overset{\frown}{FT}$ f. $m\angle ATC$

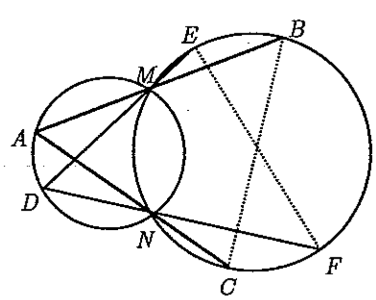


15. The circles are internally tangent at T. Prove that $\overline{AB} \parallel \overline{CD}$.



16. From point A outside a circle a secant and a tangent are drawn. The secant intersects the circle at B and C while the tangent touches the circle at D. The bisector of $\angle BDC$ intersects $\overline{BC}$ at E. Prove that $AD = AE$.

17. Two circles intersect at M and N. From point A on the left-hand circle lines are drawn through M and N intersecting the right-hand circle at B and C, respectively. Likewise, from D lines are drawn intersecting the right-hand circle at E and F. Prove that $BC = EF$.

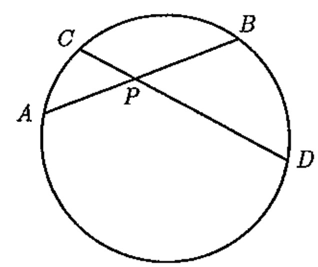


Exercises 7.7

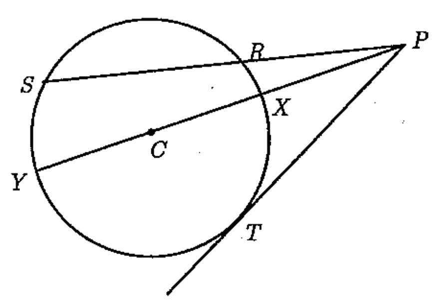
1. Prove **Theorem 7.20**: If $\overline{PAB}$ is a secant to a circle and $\overline{PT}$ is a tangent segment to the same circle, then $PA \cdot PB = (PT)^2$.

2. Prove **Theorem 7.21**: If two chords are drawn through the same point in the interior of a circle, then the product of the lengths of the segments on one chord is equal to the product of the lengths of the segments of the other.

3. Consider the figure to the right.
   a. If $AB = 6$ and $AP = 2$, what is the power of point P?
   b. If $AP = 4$, $BP = 9$, and $CP = 12$, what is DP?
   c. If $AB = 11$, $CD = 9$, and $AP = 2$, what is CP?



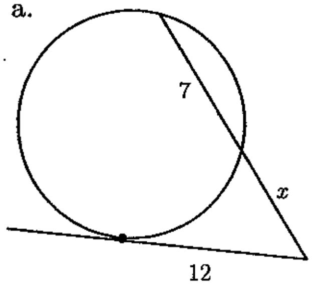
4. In the figure to the right, $\overleftrightarrow{PT}$ is tangent to $\odot C$ at point T.
   a. If $PR = 4$ and $RS = 5$, what is PT?
   b. If $PR = 6$ and $PT = 4\sqrt{3}$, what is RS?
   c. If $PR = RS = 3$ and $PX = 2$, what is PY?
   d. If $PT = 15$ and $PX = 9$, what is CX?
   e. If $PT = \frac{3}{2}$ and $CX = \frac{1}{PX}$, what is PX?



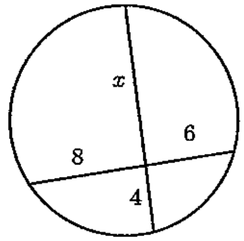
5. A circle has radius 6.
   a. Find the powers of points that are 4, 3, 2, and 1 unit from the center.
   b. What is the power of an interior point of the circle that is x units from the center? What value of x makes that number as large as possible?

6. Solve for x in each of the following.

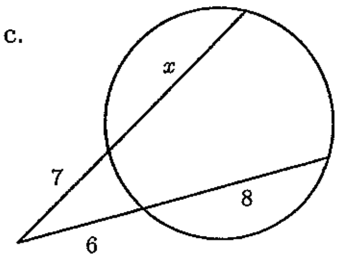
a.



b.

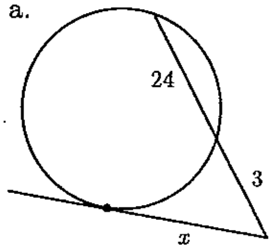


c.

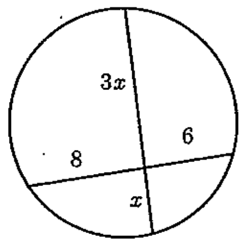


7. Solve for x in each of the following.

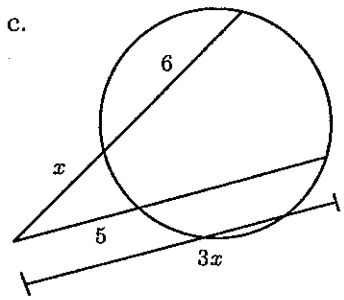
a.



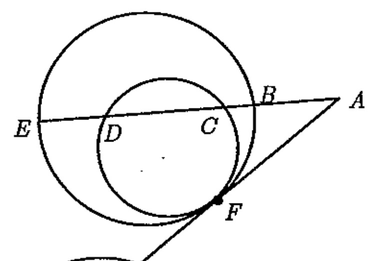
b.



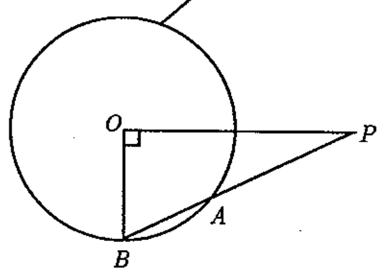
c.



8. The two circles shown are internally tangent at F. $\overline{AF}$ is the common tangent. Prove that $AB \cdot AE = AC \cdot AD$.



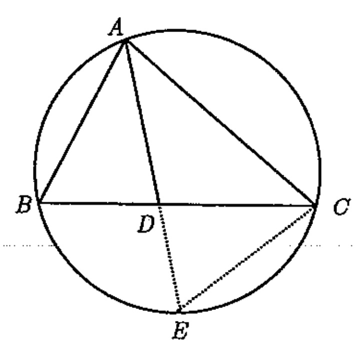
9. In $\odot O$, $\overline{PO} \perp \overline{OB}$ and PO equals the length of the circle's diameter. Determine $\frac{PA}{AB}$.



10. Given: $\overline{AD}$ bisects $\angle BAC$

Prove: $(AD)^2 = AB \cdot AC - BD \cdot CD$

[Hint: Show that $\Delta ABD \sim \Delta AEC$.]



11. In ΔABC, $AC = 4$, $BC = 5$, and $AB = 6$. $\overline{CT}$ is the bisector of $\angle ACB$.
   a. Recalling Theorem 5.4, determine the lengths AT and BT.
   b. Using the result of problem 10 above, determine CT.
   c. Explain why $m\angle ACB = 2 \cdot m\angle B$.

12. Repeat the previous problem with $AC = 9$, $BC = 7$, and $AB = 12$.

13. Prove indirectly: If B is between A and C, and if P is between A and Q, with $\overleftrightarrow{APQ}$ and $\overleftrightarrow{ABC}$ different lines, and if $AB \cdot AC = AP \cdot AQ$, then there is a circle passing through all four of the points B, C, P, and Q.

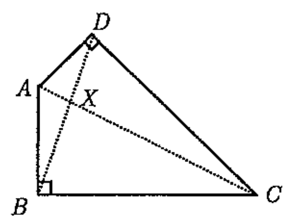
14. A tangent and a secant are drawn to a circle from the same external point. The tangent segment is two units longer than the external part of the secant and two units shorter than the internal part. How long is the tangent segment?

15. A tangent and a secant are drawn to a circle from the same external point. The tangent segment has length 10. The external part of the secant segment is as much shorter than 10 as the internal part is longer than 10. How long is the external part of the secant?

16. A tangent and a secant are drawn to a circle from the same external point. Prove that the perpendicular to the secant from the point of tangency is the geometric mean of the two perpendiculars to the tangent from the points where the secant intersects the circle.

17. A chord of a circle is 12 units long.
   a. If P is the midpoint of the chord, then what is the power of the point P?
   b. If Q is on the chord and $PQ = x$, then what is the power of the point Q in terms of x? Is it clear that the midpoint of a chord is the point on the chord for which the power of the point is largest?

18. In quadrilateral $ABCD$, $\overline{AB} \perp \overline{BC}$ and $\overline{AD} \perp \overline{CD}$.
   a. Why is $ABCD$ a cyclic quadrilateral?
   b. Why is $\angle ABD$ congruent to $\angle ACD$?
   c. Why is $AX \cdot CX$ equal to $BX \cdot DX$?



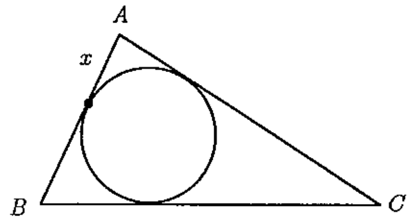
19. Chords $\overline{AB}$ and $\overline{CD}$ of a circle intersect at P. $AP = 1$ and $CP = 2$. If one of the chords has length 9, then what are the possible lengths of the other chord?

20. Chords $\overline{AB}$ and $\overline{CD}$ of a circle intersect at P. $AP = 1$ and $CP = 3$. If one of the chords has length 13, then what are the possible lengths of the other chord?

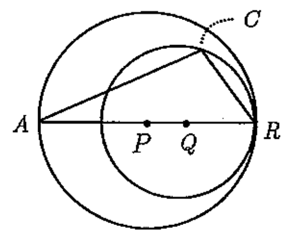
Chapter 7 Review Exercises

1. A right triangle has legs a and b and hypotenuse c. What is the radius of the inscribed circle?

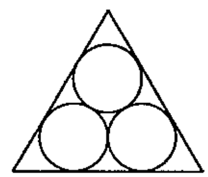
2. The circle is inscribed in ΔABC. $AB = 3$ and $AC = 6$. What integer value(s) of BC will make x an integer?



3. $\odot P$ and $\odot Q$ are internally tangent at R. Show that $\overline{AC}$ is not perpendicular to $\overline{RC}$.



4. Given three congruent and mutually tangent circles, and a circumscribed equilateral triangle, as pictured to the right. If the side of the triangle is 16, what is the radius of one of the circles?



5. Two tangents of length 24 are drawn from the same point to a circle of radius 10. What is the length of the chord joining the points of tangency?

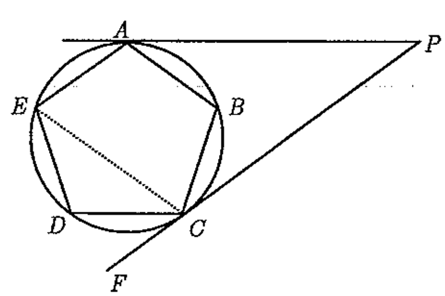
6. The bases of an isosceles trapezoid circumscribed about a circle are 24 and 8. What is the radius of the circle?

7. A circle is inscribed in a triangle whose sides are 9, 11, and 14. How far is each point of tangency from the adjacent vertices of the triangle?

8. Three circles are tangent to each other externally. The segments joining their centers are 9, 11, and 14. What are the radii of the circles?

9. $\odot P$ and $\odot Q$ are internally tangent at T, and $\odot P$ passes through point Q. $\overline{TUV}$ is a chord of $\odot Q$, with point U lying on $\odot P$. Prove that U is the midpoint of $\overline{TV}$.

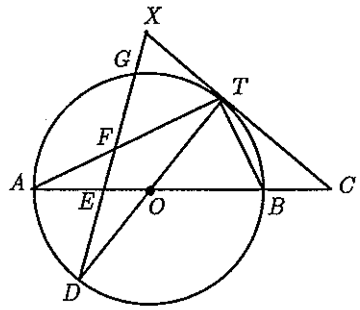
10. Regular pentagon $ABCDE$ is inscribed in a circle. From point P, tangents are drawn to the circle at A and C. Find $m\angle P$, $m\angle PAB$, and $m\angle ECF$.



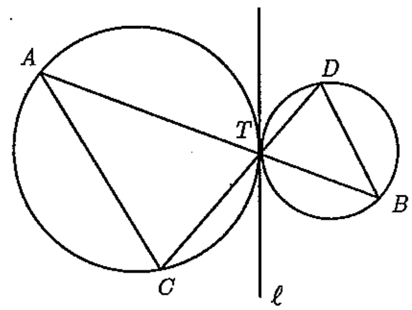
11. What is the radius of a circle circumscribed about an isosceles triangle whose sides are 10, 10, and 12?

12. In the figure, $\overline{CT}$ is tangent to $\odot O$ at T.
$m\overset{\frown}{AT} = 130$ and $m\angle D = 20$. Determine each of the following.

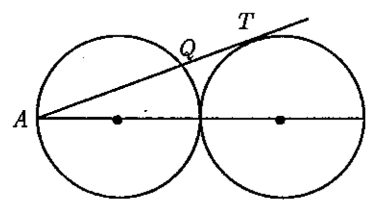
a.	$m\angle ABT$	b.	$m\angle A$
c.	$m\angle CTB$	d.	$m\angle C$
e.	$m\angle TOB$	f.	$m\overset{\frown}{AD}$
g.	$m\angle AEG$	h.	$m\angle AFG$
i.	$m\angle DTB$	j.	$m\angle X$



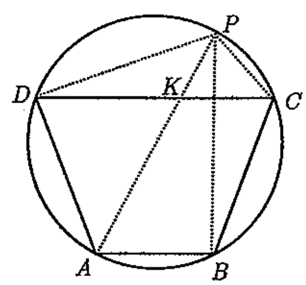
13. In the figure to the right, the circles are externally tangent to each other at T. Their common internal tangent line is ℓ. Segments $\overline{AB}$ and $\overline{CD}$ are drawn through T. Prove $\overline{AC} \parallel \overline{BD}$.



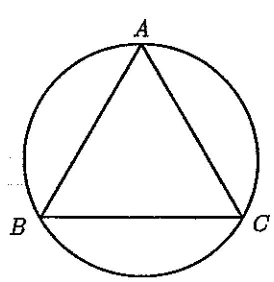
14. In the figure to the right, the circles are tangent to each other. Both circles have radius 1. $\overline{AT}$ is tangent to the right-hand circle at T. Determine the lengths of $\overline{AT}$ and $\overline{AQ}$.



15. Let $ABCD$ be a trapezoid (with $\overline{AB} \parallel \overline{CD}$) inscribed in a circle. Let P be any point on $\overset{\frown}{CD}$, and let $\overline{PA}$ intersect $\overline{CD}$ at point K. Prove that $DP \cdot BC = BP \cdot DK$.

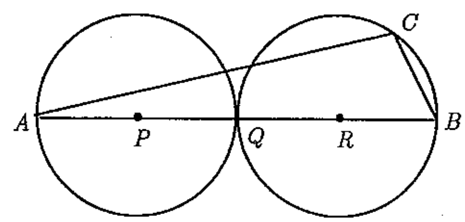


16. ΔABC is inscribed in a circle. If $AB = AC = 17$ and $BC = 16$, determine the diameter of the circle exactly.

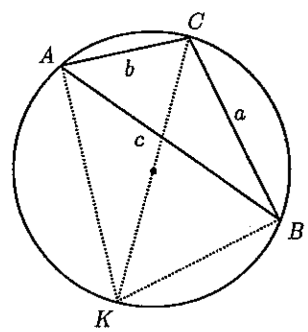


17. Quadrilateral $ABCD$ is inscribed in a circle. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at M. $\angle ABC$, $\angle BMC$, and $\angle BAD$ have measures 70, 85, and 95, respectively. Find $m\angle BAC$.

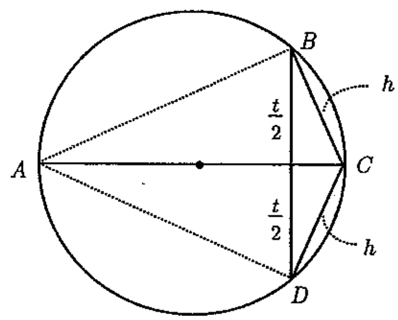
18. Congruent circles P and R are externally tangent at Q. Show that $\overline{AC}$ is not perpendicular to $\overline{BC}$.



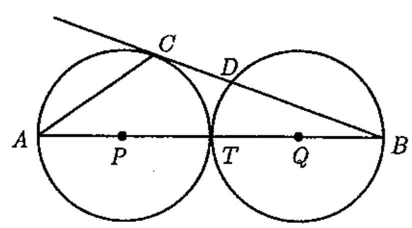
19. Let ΔABC be inscribed in a circle of radius 1, with sides having the lengths shown in the figure. Use Ptolemy's Theorem to show that $c = \frac{a}{2}\sqrt{4-b^2} + \frac{b}{2}\sqrt{4-a^2}$.



20. Chord $\overline{BD}$ cuts off a minor arc of length t in a circle of radius 1. Use Ptolemy's Theorem to show that the length of the chord that cuts off half that arc is $h = \sqrt{2-\sqrt{4-t^2}}$.



21. $\odot P$ and $\odot Q$ are congruent and externally tangent to each other at T. $\overline{BC}$ is tangent to $\odot P$ at C. Prove that $m\overset{\frown}{AC} = m\overset{\frown}{TC} + m\overset{\frown}{TD}$.



22. $\overline{AB}$ is a diameter of the circle. $\overline{AD}$ and $\overline{BF}$ are tangent to the circle. Prove that $\frac{BC}{AE} = \frac{AF}{BD}$.