Exercises 6.1:

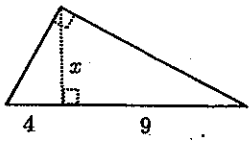
1. Prove **Theorem 6.1**: If ΔABC is a right triangle and $\overline{CD}$ is the altitude drawn to the hypotenuse, then $\Delta ABC \sim \Delta ACD \sim \Delta CBD$.

2. Prove **Corollary A to Theorem 6.1**: If ΔABC is a right triangle and $\overline{CD}$ is the altitude drawn to the hypotenuse, then $CD = \sqrt{AD \cdot DB}$.

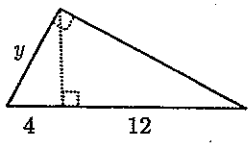
3. Prove **Corollary B to Theorem 6.1**: If ΔABC is a right triangle and $\overline{CD}$ is the altitude drawn to the hypotenuse, then $AC = \sqrt{AB \cdot AD}$ and $CB = \sqrt{AB \cdot DB}$.

4. In the figures below with the right angles as marked, determine the values of the variables.

a.

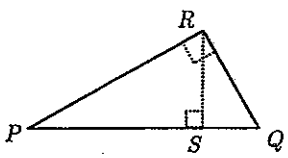


b.

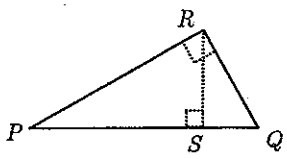


5. In the two figures below, $\overline{PR} \perp \overline{QR}$ and $\overline{SR} \perp \overline{PQ}$.

a. If $PS = 6$ and $PQ = 8$, determine PR.

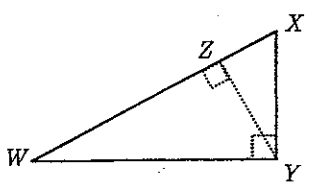


b. If $RS = 8$ and $PS = 16$, determine QS.

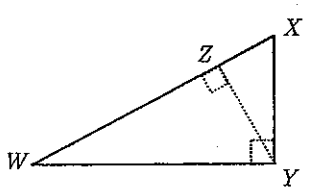


6. In the two figures below, $\overline{WY} \perp \overline{XY}$ and $\overline{YZ} \perp \overline{WX}$.

a. If $XZ = 3$ and $WZ = 24$, determine XY, ZY, and WY.



b. If $XY = 8$ and $WZ = 12$, determine XZ, YZ, and YW.

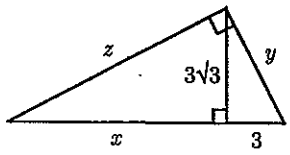


7. Determine each of the following in the figure.

a. x

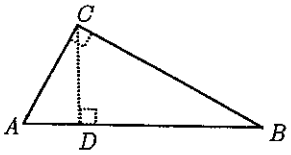
b. xy

c. $x+y+z$

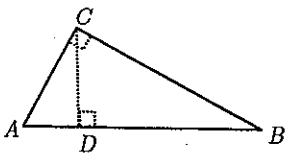


8. In each of the following, $\overline{AC} \perp \overline{CB}$ and $\overline{CD} \perp \overline{AB}$.

a. If $AD = 7$ and $AB = 11$, determine CD.

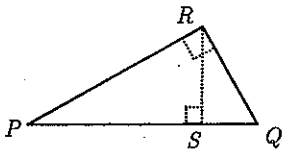


b. If $CD = 8$ and $AD = 6$, determine AB.

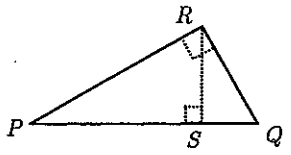


9. In each of the following, $\overline{PR} \perp \overline{QR}$ and $\overline{RS} \perp \overline{PQ}$.

a. If $PQ = 12$ and $QS = 4$, determine PR.



b. If $RQ = 7$ and $PQ = 12$, determine PS.



10. Given: $\overline{XZ}$ is an altitude in right ΔWXY.

Prove: $WZ \cdot XY = WX \cdot XZ$

11. Given: $\overline{XZ}$ is an altitude in right ΔWXY.

Prove: $WX \cdot XY = WY \cdot XZ$

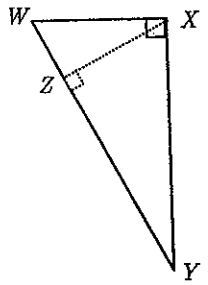
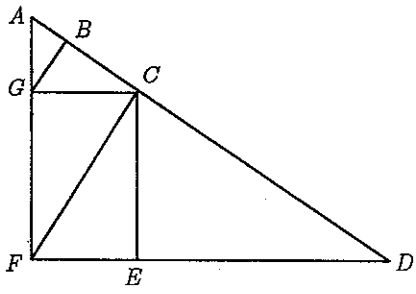


Figure for problems 10 and 11

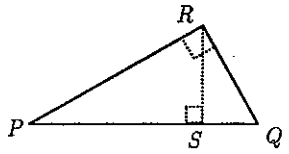
12. $\overline{PS}$ is the altitude to the hypotenuse of right ΔPQR. If $PQ = 15$ and $SR = 16$, what is the length of $\overline{QS}$?

13. In the figure, $\overline{AG} \perp \overline{GC}$, $\overline{AF} \perp \overline{FD}$, $\overline{GB} \perp \overline{AD}$, $\overline{FC} \perp \overline{AD}$, and $\overline{CE} \perp \overline{FD}$. If $AG = 6$ and $AB = 4$, determine BC, GF, and CD.



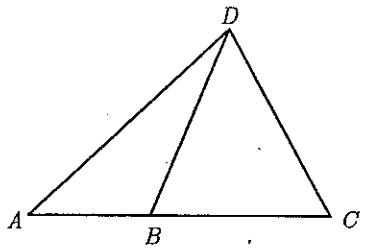
14. a. In the figure, $PR = 6$ and $QS = 5$. Determine PS and RS.

b. In the figure, $RQ = 6$ and $PS = 9$. Determine QS.

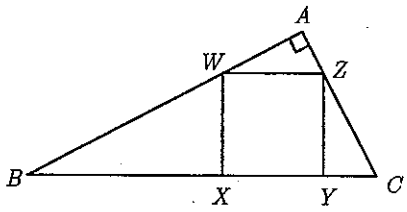


15. Given: In the figure, $\angle ADC \cong \angle DBC$.

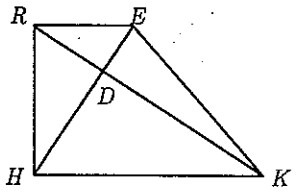
Prove: DC is the geometric mean between AC and BC.



16. A new take on a problem from earlier (problem 17 from the Review Exercises at the end of Chapter 5).
Square $WXYZ$ is inscribed in right ΔABC so that $\overline{XY}$ lies on hypotenuse $\overline{BC}$. Prove that XY is the geometric mean between BX and YC.
Draw a line through W and parallel to $\overline{AC}$.



17. Trapezoid $REKH$ has right angles at R and H. The diagonals are perpendicular to each other at D. Prove that RH is the geometric mean between RE and HK.



18. $\overline{BC}$ is the hypotenuse of right ΔABC and $\overline{AD}$ is the median to the hypotenuse. E is a point on the hypotenuse taken so that $\overline{AE} \cong \overline{AB}$. Prove that AB is the geometric mean between BE and BD.

Exercises 6.2

1. Determine (in simplest radical form) the hypotenuse of a right triangle with legs
   a. 4 and 6 b. 5 and $5\sqrt{3}$ c. 3 and 6

2. Determine the third side of a right triangle whose hypotenuse and one leg have lengths
   a. 13 and 12 b. 17 and 15 c. 29 and 21

3. Determine the third side of a right triangle whose hypotenuse and one leg have lengths
   a. $\sqrt{29}$ and 5 b. $\sqrt{38}$ and $2\sqrt{5}$ c. $5\sqrt{3}$ and $\sqrt{3}$

4. Find two right triangles with integer lengths for both legs and having a hypotenuse of length 25.

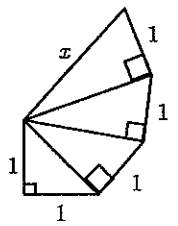
5. Determine two right triangles with a hypotenuse of length 10.

6. In right ΔABC, $AB = 5$ and $AC = 7$. What are the two possible lengths for side $\overline{BC}$?

7. Determine the perimeter of a rhombus with diagonals 12 and 16.

8. Cal Q. Later walked 2 miles north, 6 miles west, then 4 miles north, and 2 miles west. If Cal decides to go "straight," how far must he walk to get back to his starting point?

9. Determine x in the partial spiral to the right.

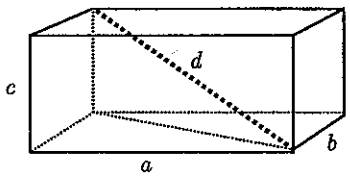


10. Determine the length of the altitude to the base of an isosceles triangle with sides of length
    a. 5, 5, and 6; b. 25, 25, and 48; c. 41, 41, and 18

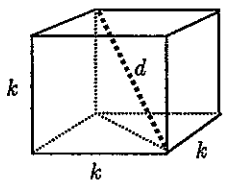
11. Determine the altitude of an isosceles trapezoid whose sides are
    a. 10, 30, 10, and 20 b. 6, 8, 8, and 12

12. An isosceles trapezoid has bases of length 6 and 16 and legs of length 13. Determine the altitude to a base and the length of the diagonals.

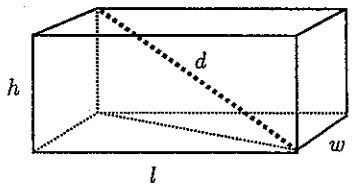
13. The dimensions of a rectangular box are given below. Determine the length of the diagonal d.
    a. $a = 7,\ b = 6,\ c = 6$
    b. $a = 6,\ b = 6,\ c = 3$
    c. $a = 5,\ b = 5,\ c = 2$
    d. $a = 3,\ b = 2,\ c = 1$



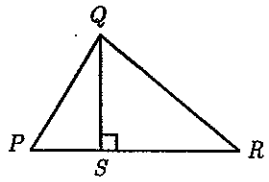
14. A cube is a rectangular box (as in the last problem) with a, b, and c all the same. Find the length of the diagonal d for a cube with edge
    a. 6 b. k



15. For the general version of problem 13, use a box with length l, width w, and height h as pictured. Find the space diagonal d of the box.

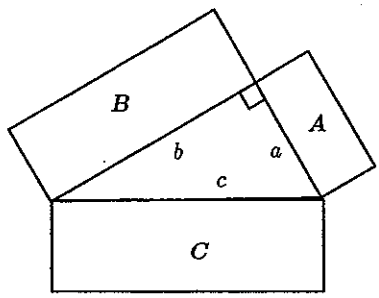


16. In the figure, $PQ = 13$, $QR = 15$, and $QS = 12$. Determine PR.

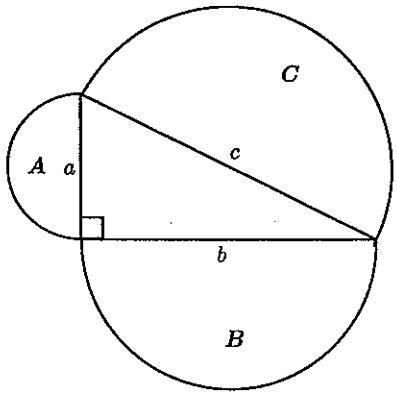


17. The sides of a triangle have lengths 7, 8, and 9. Determine the length of the altitude to the side of length 8.

18. In the figure to the right, *Region* ***A***, *Region* ***B***, and *Region* ***C*** are rectangles whose short side is one-third as long as the long side and whose long side lies on a side of the right triangle.
Show that *Area* ***A*** + *Area* ***B*** = *Area* ***C***.

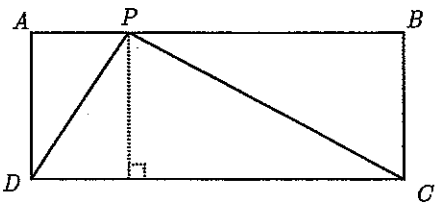


19. In the figure at the right, *Region* ***A***, *Region* ***B***, and *Region* ***C*** are semicircles. Using the fact that the area of a circle is given by πr^2 (where r is the radius), show that *Area* ***A*** + *Area* ***B*** = *Area* ***C***.

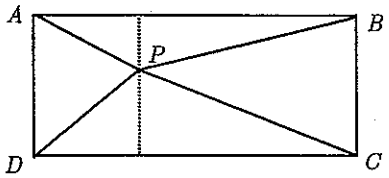


20. Quadrilateral $ABCD$ is a rhombus. Prove that $(2 \cdot AB)^2 = AC^2 + BD^2$.

21. Prove that if point P is on side $\overline{AB}$ of rectangle $ABCD$, then $(PA)^2 + (PC)^2 = (PB)^2 + (PD)^2$.



22. Prove that if point P is in the interior of rectangle $ABCD$, then $(PA)^2 + (PC)^2 = (PB)^2 + (PD)^2$.



Exercises 6.3

1. Show that the triangle with sides of the given lengths is a right triangle.
   a. 15, 20, 25 b. 36, 48, 60 c. $\sqrt{5}$, $\sqrt{5}$, $\sqrt{10}$ d. 4, 8, $4\sqrt{3}$

2. In ΔABC, $AB = \sqrt{20}$ and $AC = \sqrt{80}$. If the altitude from A to $\overline{BC}$ is 4, prove that $\angle A$ is a right angle.

3. If one side of a parallelogram is 10 and the diagonals are 12 and 16, prove the diagonals are perpendicular to each other.

4. Given that D is the midpoint of hypotenuse $\overline{AC}$ of right triangle ΔABC and E is the midpoint of $\overline{AB}$, prove that $\overline{ED} \perp \overline{AB}$.

5. Consider the following list of Pythagorean triples.

n	a	b	c
1	3	4	5
2	5	12	13
3	7	24	25
4			
5			

   a. Fill in rows 4 and 5.
   b. Find the three numbers that belong in row n. (That is, find formulas for a, b, and c in terms of n.)

6. The lengths of the sides of a triangle are given. In each case, determine if the triangle is a right triangle, an acute triangle, or an obtuse triangle.
   a. 7, 9, 10 b. 3, 6, 8 c. 8, 10, 12
   d. 1, 3, $\sqrt{5}$ e. 8, 15, 17 f. $\sqrt{2}$, 2, $\sqrt{2}$

7. True or False? A triangle whose sides are 9, 12, and 15 can be placed next to a right triangle whose sides are 12, 16, and 20 so that the resulting figure is a right triangle. Justify your answer.

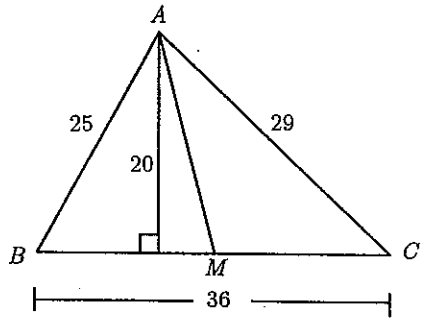
8. Consider the following list of Pythagorean triples in which every other row has three numbers divisible by 2.

n	a	b	c
1	4	3	5
2	6	8	10
3	8	15	17
4	10	24	26
5			
6			

a. Fill in rows 5 and 6.

b. Find the three numbers that belong in row n. (That is, find formulas for a, b, and c in terms of n.)

9. Consider a triangle whose sides have lengths 25, 29, and 36. The altitude to the side of length 36 has length 20, as indicated in the figure. What is the length of the **median** $\overline{AM}$ to the side of length 36?



10. Consider a right triangle with legs 72 and 96 (both of which are divisible by 24).

a. What family of right triangles does this triangle belong to?

b. What is the hypotenuse of this right triangle?

11. Let a number m be given. Consider the three numbers defined in terms of m by $a = m$, $b = \frac{1}{2}\left(m^2 - 1\right)$, and $c = \frac{1}{2}\left(m^2 + 1\right)$. Show that these numbers satisfy the Pythagorean relationship $a^2 + b^2 = c^2$, That is, show that the following formula is true for all values of m: $m^2 + \left(\frac{1}{2}\left(m^2 - 1\right)\right)^2 = \left(\frac{1}{2}\left(m^2 + 1\right)\right)^2$

12. Using the formula from problem 11, determine the Pythagorean Triples for

a. $m = 5$ b. $m = 7$ c. $m = 4$ d. $m = 6$

Exercises 6.4

1. Determine the length of the hypotenuse of an isosceles right triangle whose legs measure
   a. 2 b. 10 c. $\sqrt{2}$ d. $\sqrt{18}$

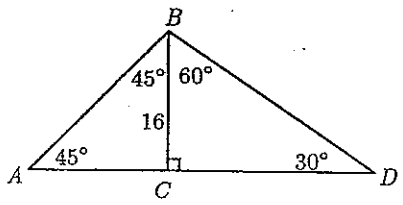
2. Determine the side of a square whose diagonal measures
   a. 4 b. 1 c. 20 d. $6\sqrt{2}$ e. D

3. Determine the length of the shorter leg of a 30–60–90 triangle whose longest leg measures
   a. 1 b. 2 c. $\sqrt{3}$ d. $\sqrt{15}$

4. Determine the length of the hypotenuse of a 30–60–90 triangle whose longest leg measures
   a. 1 b. 2 c. $\sqrt{3}$ d. $\sqrt{15}$

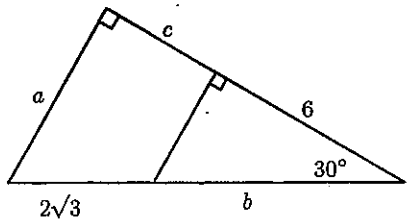
5. Determine the length of the longest leg of a 30–60–90 triangle whose shortest leg measures
   a. 1 b. 2 c. $\sqrt{3}$ d. $\sqrt{15}$

6. Use the diagram at the right to determine AC, CD, BD, and AB.

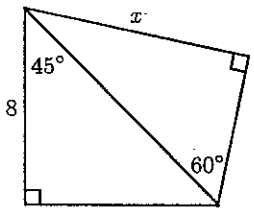


7. Determine the diagonal of a cube if each edge measures
   a. 2 b. 3 c. x

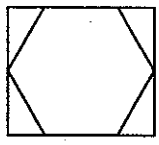
8. Using the figure at the right, determine a, b, and c.



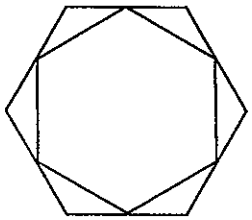
9. Two right triangles are aligned as shown. Determine x.



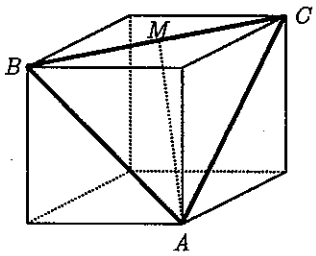
10. Pictured is a regular hexagon with a rectangle fitting around it. If the side of the hexagon is 6, then what are the dimensions of the rectangle?



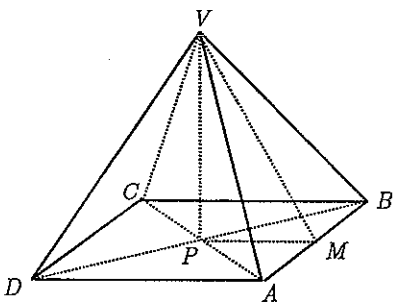
11. The large hexagon is regular. The midpoints are joined in order to form a second regular hexagon inscribed within the first.
    a. If the first hexagon's side is 8, what is the side of the inscribed hexagon?
    b. If the first hexagon's side is k, what is the side of the inscribed hexagon?



12. Pictured is a cube of side 8. The vertices A, B, and C are joined to form a triangle.
    a. Is the triangle necessarily equilateral?
    b. What is the distance from A to the midpoint of $\overline{BC}$?



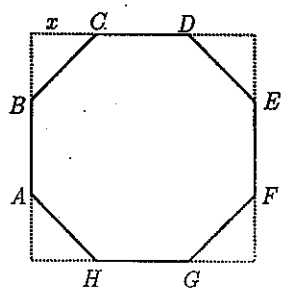
13. Pictured is a pyramid with a square base $ABCD$. ΔVAB, ΔVBC, ΔVCD, and ΔVDA are all equilateral triangles with $AB = 12$. $\overline{VP} \perp$ (plane of $ABCD$).
    M is the midpoint of $\overline{AB}$.
    Determine each of the following.
    a. AC b. VM
    c. MP d. VP



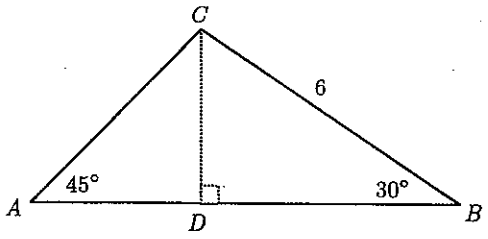
14. $ABCDEFGH$ is a regular octagon.

a. If $AB = 4$, then what is x?

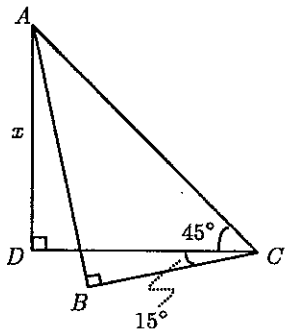
b. If $x = 6$, then what is the perimeter of $ABCDEFGH$?



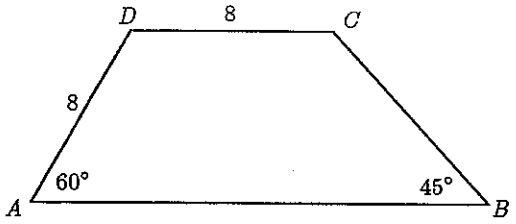
15. Determine the perimeter of the triangle shown to the right.



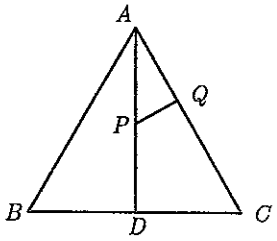
16. Determine the **exact** value of x in the figure, given that $AB = 12$.



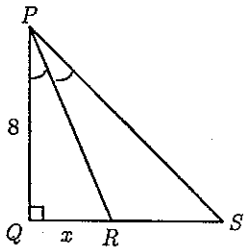
17. What is the perimeter of the trapezoid $ABCD$ pictured?



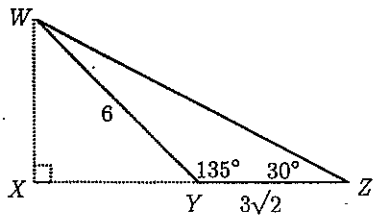
18. ΔABC is equilateral with side 12. Point P is the midpoint of altitude $\overline{AD}$, and $\overline{PQ} \perp \overline{AC}$. What is the length of $\overline{CQ}$?



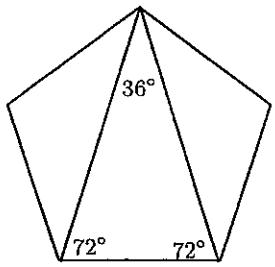
19. $\overline{PR}$ is the bisector of one of the acute angles of a 45–45–90 triangle, as shown to the right. If $PQ = 8$, then what is the value of x?

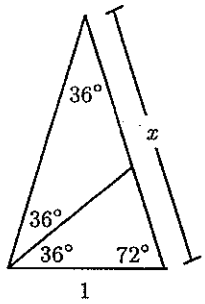


20. Show that ΔWYZ pictured to the right is impossible.

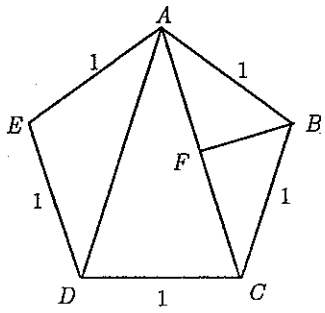


21. Consider the 18–72–90 special right triangle. This is one-half of a 36–72–72 isosceles triangle pictured to the right, which occurs in consideration of a regular pentagon. Let the base of the 36–72–72 triangle be 1. Determine the exact value of each of the following lengths.
    a. The length x of the longer side of the triangle
    b. The altitude to the base of the triangle





22. Use the result of problem 21 to deal with the 36–54–90 special right triangle. In the regular pentagon $ABCDE$ to the right, drop altitude $\overline{BF}$ from B to $\overline{AC}$. With the side of the pentagon being 1, what are the exact lengths of $\overline{AF}$ and $\overline{BF}$?



Exercises 6.5

1. Remembering the relationship 30–60–90 $\Leftrightarrow$ $\left(x, x\sqrt{3}, 2x\right)$, determine each of the following exactly.

   a. $\sin 30°$ b. $\cos 30°$ c. $\tan 30°$

   d. $\sin 60°$ e. $\cos 60°$ f. $\tan 60°$

2. Remembering the relationship 45–45–90 $\Leftrightarrow$ $\left(x, x, x\sqrt{2}\right)$, determine each of the following exactly.

   a. $\sin 45°$ b. $\cos 45°$ c. $\tan 45°$

3. In an isosceles trapezoid with sides 5, 10, 17, and 10, determine each of the following:
   a. the sine of one of the acute angles;
   b. the cosine of one of the acute angles;
   c. the tangent of one of the acute angles;
   d. the measure of one of the acute angles in degrees.

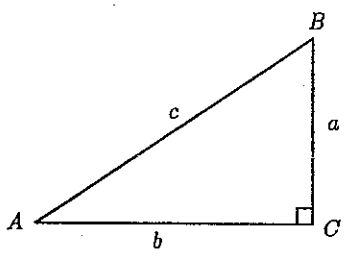
4. Using the triangle at the right and the definitions of the trigonometric ratios, verify the following.

   a. $(\sin A)^2 + (\cos A)^2 = 1$

   b. $\frac{a}{\sin A} = \frac{b}{\sin B}$

   c. $\frac{\sin A}{\cos A} = \tan A$

   d. $\sin A = \cos(90 - A)$



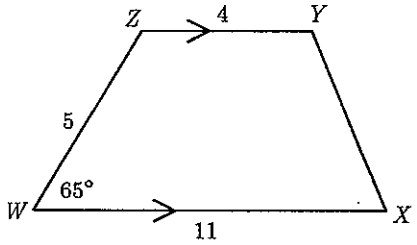
5. Rhombus $PQRS$ has a perimeter of 60 and one diagonal is 15. Determine two possible values for $\sin \angle PQS$.

6. In right ΔABC, $AB = 16$, $BC = 30$, and $AC = 34$. Find, to the nearest degree, the acute angles of the triangle.

7. The legs of an isosceles triangle are each 16. If the base is 12 determine:
   a. the base angles to the nearest degree;
   b. the exact length of the altitude to the base.

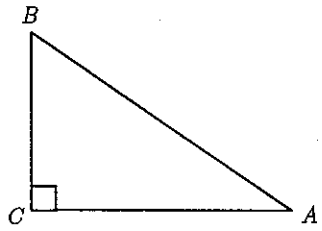
8. Determine the perimeter of a trapezoid $WXYZ$ in which $\overline{YZ} \parallel \overline{WX}$, $\cos Y = \frac{\sqrt{3}}{2}$, and $WZ = WX = XY = 2$.

9. Consider trapezoid $WXYZ$ pictured to the right.
   a. Determine the length of $\overline{XY}$ (to the nearest 0.01).
   b. Determine $m\angle X$ (to the nearest degree).



10. As an angle increases in size from 0° to 90°,
    a. describe how the sine of the angle changes;
    b. describe how the cosine of the angle changes;
    c. describe how the tangent of the angle changes.

11. In the right triangle pictured, $m\angle A = 35$. Give answers to the nearest 0.01.
    a. If $AB = 5$, what is AC?
    b. If $AC = 5$, what is BC?
    c. If $BC = 7$, what is AC?
    d. If $BC = 7$, what is AB?

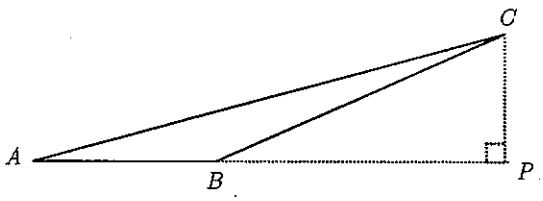


12. Right ΔXYZ has $XZ = 3$, $YZ = 4$, and $XY = 5$. Write the three trigonometric ratios for $\angle X$ and $\angle Y$.

13. In ΔXYZ, $m\angle X = 42$ while $XY = 7.2$ and $XZ = 12.5$. Determine the length of the altitude from Y to $\overline{XZ}$ and then the length of $\overline{YZ}$.

14. In right ΔPQR, the right angle is at vertex P and $\tan Q = 5$. If $QR = 7$, then what is PQ?

15. In ΔABC, $AB = 7$, $BC = 12$, and $m\angle ABC = 140$.
    a. Find the lengths of $\overline{CP}$ and $\overline{PB}$.
    b. Find the length of $\overline{AP}$.
    c. Determine $m\angle A$.
    d. Determine the length of $\overline{AC}$.



16. $ABCDEFGHI$ is a regular nonagon (a 9-sided polygon) inscribed in a circle whose center is at point P. Among other things, this means that each of the vertices of $ABCDEFGHI$ is the same distance from P (the radius of the circle) and that each of the "central angles" $\angle APB$, $\angle BPC$, $\angle CPD$, ... , $\angle IPA$ has the same measure. In this problem, the radius of the circle is 6.

a. What is the measure of each of the central angles?

b. What is the distance from point A to radius $\overline{PB}$? (Round to the nearest 0.01.)

c. What is $m\angle ABP$?

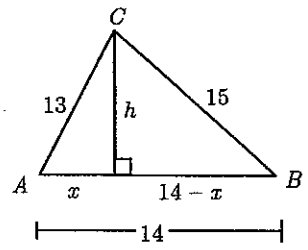
d. What is the length of side $\overline{AB}$ of nonagon $ABCDEFGHI$ (to the nearest 0.01)?

e. What is the perimeter of nonagon $ABCDEFGHI$ (to the nearest 0.01)?

17. In ΔABC, the sides have lengths 13, 14, and 15, as shown to the right. An altitude to the side of length 14 is drawn, creating two right triangles.

a. Use the Pythagorean Theorem in each of those triangles to help determine x and h.

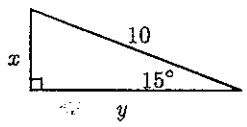
b. To the nearest 0.1 degree, what are the angles of the triangle?



Exercises 6.6

1. ΔABC is isosceles with $AC = CB = 20$ and $m\angle A = m\angle B = 68$. $\overline{CD}$ is an altitude.
   a. Determine the length of altitude $\overline{CD}$ to the nearest .01.
   b. Determine the length of $\overline{AB}$ to the nearest .01.

2. Determine the values of x and y in the figure to the nearest 0.01.



3. Looking from a cliff 300 meters above the water, the angle of depression to a ship is $32°$.
   a. How far is the ship from the top of the cliff (nearest 0.01 m)?
   b. How far is the ship from shore (nearest 0.01 m)?

4. An escalator at Logan airport is 80 feet long. If it rises 30 feet vertically, determine the angle it makes with the horizontal (nearest 0.1 degree).

5. Determine the height of a flagpole that casts a shadow 45 feet long when the angle of elevation of the sun is 35°. Give your answer to the nearest 0.01 ft.

6. Determine the angle of elevation of the sun when a flagpole 37 feet tall casts a shadow
   a. 30 feet long; b. 37 feet long; c. 50 feet long.

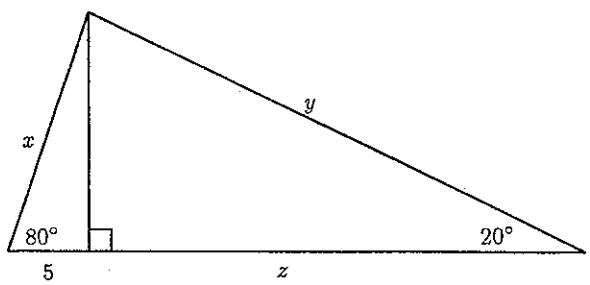
7. A pilot of a Navy jet is cruising at 3000 feet and sights the aircraft carrier she is to land on at an angle of depression of 15°. How far is the plane from the carrier?

8. Before the Memorial Bell Tower was fixed, it was said that geometry students could drop a rock from the top of the 130-foot tall structure and it would land 15 feet from the base. What was the angle the Bell Tower leaned off from the vertical?

9. How high is a church spire that casts a 90-foot shadow at the same time that a 5-foot gate post casts a 3-foot shadow; and what is the angle of elevation of the sun?

10. Standing at the top of a lighthouse 200 feet high, a lighthouse keeper sighted both an airplane and a ship that was directly beneath the plane. The angle of elevation of the plane was 25° and the angle of depression of the ship was 32°.
    a. Determine the distance of the boat from the lighthouse.
    b. Determine the height of the plane above the water.

11. Determine x, y, and z in the figure.



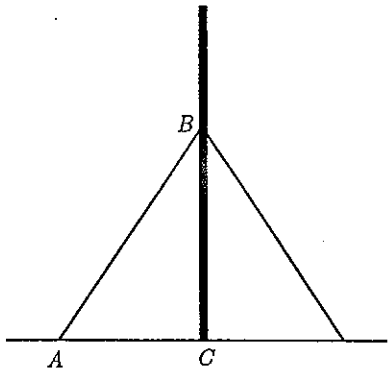
12. From the second-floor window of Commons 80 feet above the ground you watch your friend walking directly towards you. Your friend's angle of depression changes from 43° to 66°.
    a. How far has your friend walked towards Commons?
    b. How much closer to you (along the sight-lines) is your friend at the second sighting?

13. A building 200 feet tall is at the east end of a field and a building 100 feet tall is at the west end of the field. A person standing almost halfway between the buildings notes that the angles of elevation to the tops of the two buildings are 59° and 73°.
    a. How far apart are the buildings (to the nearest 10 feet)?
    b. Using your distance from part (a), what is the angle of depression (to the nearest degree) from the top of the taller building to the top of the shorter one?

14. Two buildings are on opposite sides of a street 50 meters apart. From the top of the taller building, which is 200 meters high, the angle of depression to the top of the shorter building is 20°. Determine the height of the shorter building.

15. An antenna is mounted at the edge of a roof of a house that is 30 feet tall. From the street 100 feet from the base of the house the angle of elevation to the top of the antenna is 25°. How long is the antenna?

16. A cell phone tower is anchored by long cables called guy wires such as $\overline{AB}$ in the figure. A is 175 feet from the base of the tower and $m\angle BAC = 55$.
    a. How long is the guy wire?
    b. How far above ground is the guy wire attached to the tower?
    c. After sighting the angle of elevation to the top of the tower to be 70° from point A, how tall is the tower?



17. An airplane is approaching an airport at an altitude of 28,000 feet. For the comfort of the passengers, it is best to descend at a constant angle of 8° while preparing for landing. How far from the runway should the descent begin?

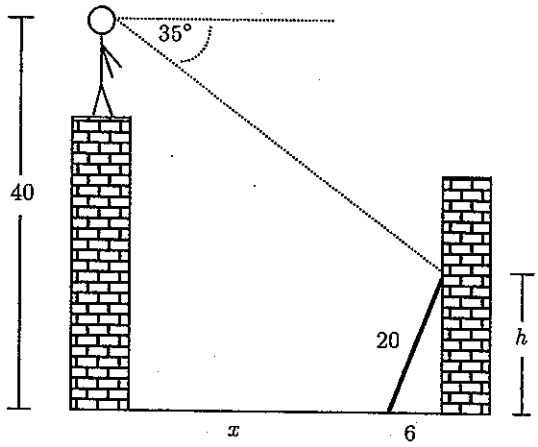
18. In regular octagon $ABCDEFGH$, diagonals $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, and $\overline{DH}$ all intersect at M. Point M is 8 units from each of the vertices of the octagon.
    a. Determine $m\angle AMB$.
    b. How far is M from $\overline{AB}$?
    c. Determine the length of a side of the octagon.

19. Let $ABCDEFGHIJ$ be a regular decagon (10-sided polygon) of side 6.
    a. What is the measure of each exterior angle of the decagon?
    b. Extend side $\overline{AB}$ through B and draw $\overline{CP}$ perpendicular to $\overleftrightarrow{AB}$ at P. How long are $\overline{BP}$ and $\overline{CP}$?
    c. What is the length of the diagonal $\overline{AC}$?

20. From the top of a building (with a flat roof), a rope 100 feet long reaches the ground 28 feet away from the base of the building.
    a. How tall is the building?
    b. What angle does the rope make with the ground?

21. As seen from the top of Day Hall, the angle of elevation to the top of the Bell Tower is 11° and the angle of depression to its base is 8°. If the Bell Tower is 100 feet high, then how far is Day Hall from the Bell Tower?

22. A 20-foot ladder is leaning up against a building, with its base 6 feet from the building, as shown. You are standing on top of a nearby building, with your eyes 40 feet above the ground level. When you look across at the ladder, the angle of depression of the top of the ladder is 35°. To the nearest 0.01 foot, what are the values of x and h? [Hint: Draw a horizontal line from the top of the ladder to the left-hand building.]

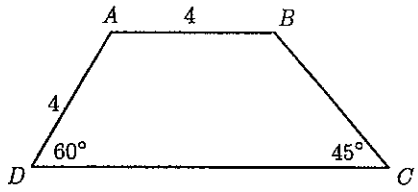


Chapter 6 Review Exercises

1. The altitude to the hypotenuse of a right triangle has length 6 and the two segments it cuts off on the hypotenuse have lengths that differ by 5. What is the perimeter of the triangle?

2. The diagonals of a rectangle are 14 inches long and intersect in a 40° angle. Determine the length and width of the rectangle.

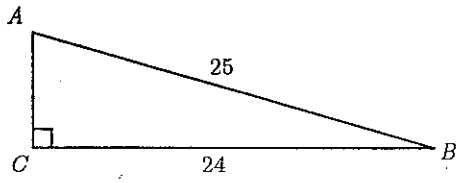
3. Determine the perimeter of the trapezoid pictured (with $\overline{AB} \parallel \overline{CD}$).



4. Two sides of a right triangle are 7 and 9. What are the possible lengths of the third side?

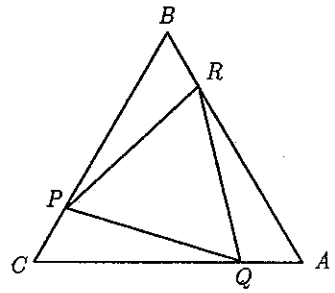
5. Determine the perimeter of trapezoid $ABCD$ in which $\overline{AB} \parallel \overline{CD}$ and $\cos \angle A = \frac{1}{2}$ if $AD = DC = CB = 4$.

6. Use the figure to find each of the following.
   a. $\sin \angle A$ b. $m\angle A$
   c. $\sin \angle B$ d. $m\angle B$



7. Points A, B, and C are three consecutive vertices of a regular octagon whose sides are 8 units long. How long is the diagonal $\overline{AC}$? Do the problem exactly, and then confirm your answer with trigonometry.

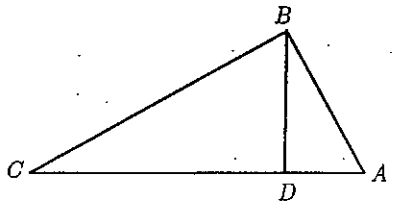
8. ΔPQR is an equilateral triangle with its vertices on the sides of equilateral ΔABC. If $PC = 2$ and $CQ = 8$, then what is the perimeter of ΔPQR? [Hint: Drop a perpendicular from P to $\overline{AC}$.]



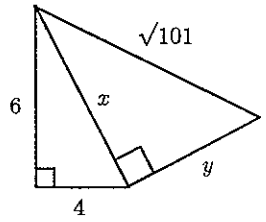
9. a. Prove that a triangle with sides of lengths 20, 21, and 29 is a right triangle.
   b. Use this triangle to **approximate** $\cos 45°$ (which we know is $\frac{\sqrt{2}}{2}$), getting both an over-estimate and an under-estimate.
   c. How close is the average of the two approximations in (b) to $\frac{\sqrt{2}}{2}$?

10. In a rhombus, if each side has length 9 and one diagonal has length 14, determine the length of the other diagonal.

11. In the figure, $\angle ABC$ and $\angle ADB$ are both right angles. $BC = 15$ and $CD = 12$. Determine AD and BD.

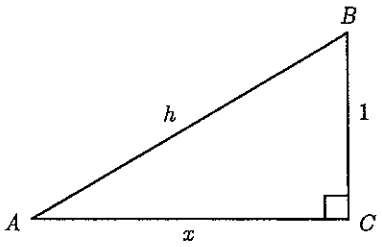


12. Determine x and y in the figure to the right.

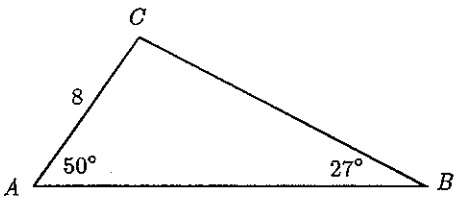


13. In the figure shown to the right, side $\overline{BC}$ stays fixed with a length of 1, while point A moves to the right toward point C (thereby changing the lengths x and h). Determine whether each of the following quantities gets larger, gets smaller, or stays the same.

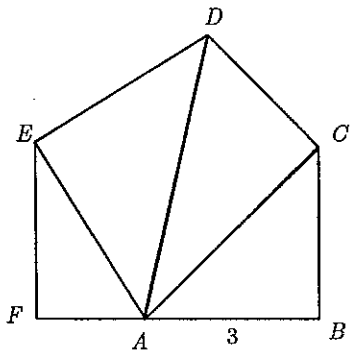
a. $m\angle A$ b. h c. $\sin\angle A$
d. $\cos\angle A$ e. $\tan\angle A$



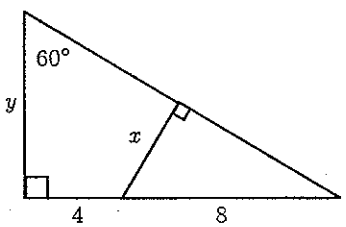
14. Determine the length AB in the triangle to the right. Give your answer correct to the nearest 0.01.



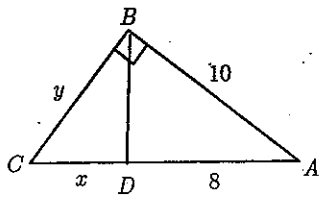
15. In the figure, $AB = 3$.
ΔABC is a 45–45–90 triangle with the right angle at B.
ΔACD is a 30–60–90 triangle with the right angle at C and the 30° angle at A.
ΔADE is a 45–45–90 triangle with the right angle at E.
ΔAEF is a 30–60–90 triangle with the right angle at F and the 30° angle at E.
Determine EF.



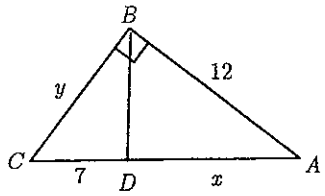
16. Determine x and y.



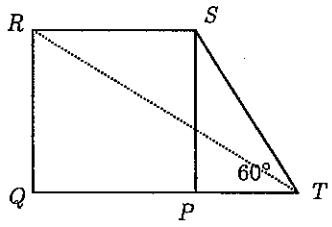
17. ΔABC is a right triangle with altitude $\overline{BD}$. Determine x and y exactly.



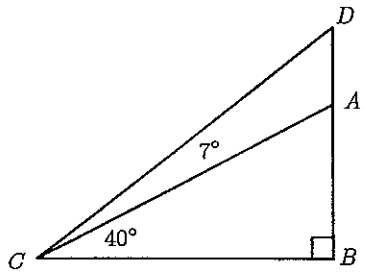
18. ΔABC is a right triangle with altitude $\overline{BD}$. Determine x and y exactly.



19. $PQRS$ is a square. $m\angle STP = 60$. Determine $m\angle RTQ$ to the nearest 0.1°.

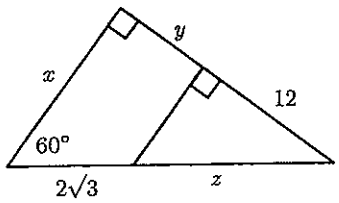


20. In the figure, $AC = 20$. Determine the length of $\overline{AD}$ to the nearest 0.01.



21. The shortest side of a triangle has length x. The next shortest side has length 24. If the longest side has length $3x + 4$, find all values of x so that the triangle is a right triangle.

22. Find **exact** values for x, y, and z.



23. Find in exact radical form the length of the diagonal of a cube of side 2.

24. $PQRSTU$ is a regular hexagon of side 6. Determine the total length of the three diagonals $\overline{PR}$, $\overline{PS}$, and $\overline{PT}$ that can be drawn from vertex P.

25. $ABCDEFGH$ is a regular octagon of side 4. Given that $ABCD$ is a trapezoid, find, to the nearest 0.01, the total length of the five diagonals $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$, and $\overline{AG}$ that can be drawn from vertex A.