5.1 Exercises

1. Solve for x:

   a. $\frac{x+3}{4x} = \frac{14}{35}$ b. $\frac{x+3}{2} = \frac{2x-1}{3}$

   c. $\frac{x}{x-3} = \frac{x+4}{x}$ d. $\frac{x-2}{x-5} = \frac{2x+1}{x-1}$

2. Let a, b, c, and d be four positive numbers for which $\frac{a}{b} = \frac{c}{d}$.

   a. Show that $\frac{a}{c} = \frac{b}{d}$.

   b. Show that $\frac{b}{a} = \frac{d}{c}$.

3. If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{a-b}{b} = \frac{c-d}{d}$.

4. a. Prove that the proportion $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a}{b} = \frac{a+c}{b+d}$. (Hint: Let $\frac{a}{b} = r$; then $a = br$ and $c = qr$. Therefore, $\frac{a+c}{b+d} = ?$)

   b. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+c+e}{b+d+f} = ?$

5. Determine the ratio of x to y in each of the following.

a. $5x = 2y$ b. $x = \frac{by}{c}$

c. $3x - 2y = 5x + 6y$ d. $\frac{4x+8y}{3x+y} = \frac{x+y}{x-y}$

6. a. If $\frac{x}{y} = \frac{12}{11}$, then find the value of $\frac{x+y}{y}$.

b. If $\frac{x}{y} = \frac{12}{11}$, then find the value of $\frac{x+12}{y+11}$.

7. What is the ratio of the total number of diagonals of a hexagon to the measure of each exterior angle of a regular decagon?

8. Given two squares with sides of 5 and 7, what is the ratio of their perimeters? Their areas?

9. The length of a segment is 10. If it is separated into two parts whose lengths are in the ratio of $3:2$, then how long is each of the two parts?

10. The ratio of the lengths of the sides of a triangle is $3:4:5$. If the perimeter of the triangle is 36, then find the length of each side of the triangle.

11. The sum of four numbers is 771. The ratio of the first number to the second is $2:3$. The ratio of the second to the third is $5:4$. The ratio of the third to the fourth is $5:6$. Find the four numbers.

12. Determine the ratio $x:y$ for each of the following.

a. $mx + nx = py$ b. $ax - ay = bx + by$ c. $ax - by = 4x + 3y$

13. a. Determine the geometric mean of 3 and 12.
b. Determine the geometric mean of 7 and 14.
c. 3 is the geometric mean of x and 12. What is x?
d. The geometric mean of $12x$ and $15x$ is $72\sqrt{5}$. What is x?

14. The ratio of the length to the width of a rectangle is $3:2$. Which of these other ratios is also $3:2$?

a. perimeter of rectangle : length of rectangle

b. area of square on long side : area of rectangle

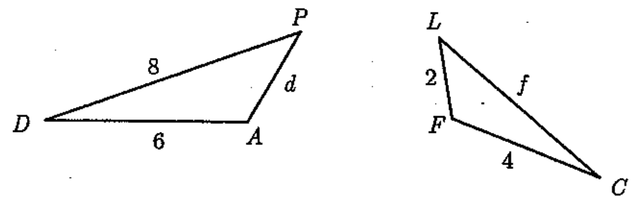
c. area of square on long side : area of square of short side

5.2 Exercises

1. Given that $\Delta ABC \sim \Delta PZT$.
   a. Which angle of ΔPZT is congruent to $\angle BCA$?
   b. Find a fraction equal to $\frac{AB}{AC}$.
   c. Identify two fractions that are equal to $\frac{ZT}{BC}$.

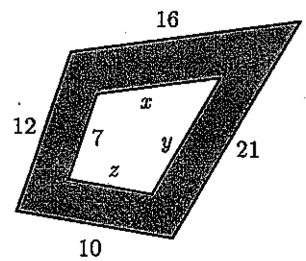
2. Given that $ABCD \sim ZYXW$.
   a. Which angle of $ZYXW$ is congruent to $\angle C$?
   b. Fill in the blanks: $\frac{AB}{ZY} = \frac{BC}{?} = \frac{?}{XW} = \frac{?}{?}$

3. Given: $\Delta DAP \sim \Delta CFL$.
   Determine the lengths PA and LC.

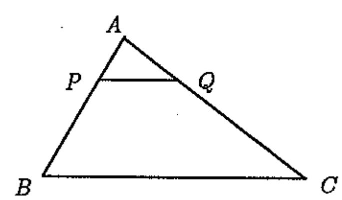


4. Two similar triangles have a pair of corresponding sides of lengths $3\frac{1}{2}$ and $8\frac{3}{4}$, respectively. What is the ratio of their perimeters?

5. If the inner and outer boundaries of the shaded region are similar quadrilaterals, find x, y, and z.

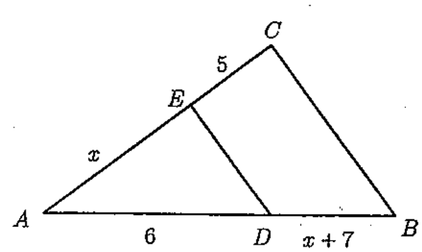


6. In the figure, $\Delta APQ \sim \Delta ABC$. If $AP = 3$, $PQ = 7$, and $AB = 10$, then find BC.

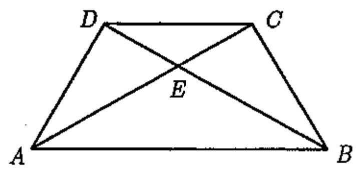


7. Which of the following statements are true?
   a. All equilateral triangles are similar.
   b. All rectangles are similar.
   c. All equilateral quadrilaterals are similar.
   d. All squares are similar.
   e. All isosceles triangles are similar.
   f. All regular hexagons are similar.

8. In the figure, $\Delta ABC \sim \Delta ADE$. Write a proportion and solve for x.



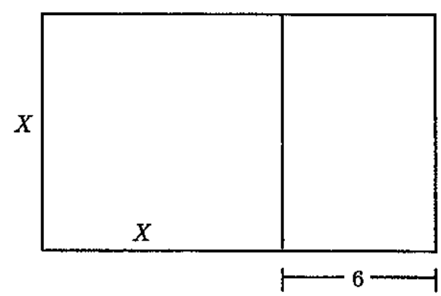
9. Given trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$ and $\Delta AEB \sim \Delta DEC$.
Prove that $AD = BC$.



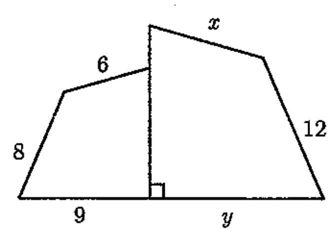
10. Square $ABCD$ has a side of 3 and square $PQRS$ has a side of 5. Find each of the following ratios in simplest form.
 a. A side of $ABCD$ to the perimeter of $ABCD$.
 b. The perimeter of $ABCD$ to the perimeter of $PQRS$.

11. $\Delta ABC \sim \Delta PQR$ with a scale factor of $2:5$. If $PQ = 7$, then find AB.

12. The large rectangle shown is a **golden rectangle**. This means that when a square is cut off, the remaining rectangle is similar to the original rectangle.
 a. What is the value of X in the figure? That is, how wide is the original rectangle?
 b. The ratio of the length to the width of a golden rectangle is called the **golden ratio**. Find the golden ratio to the nearest hundredth.



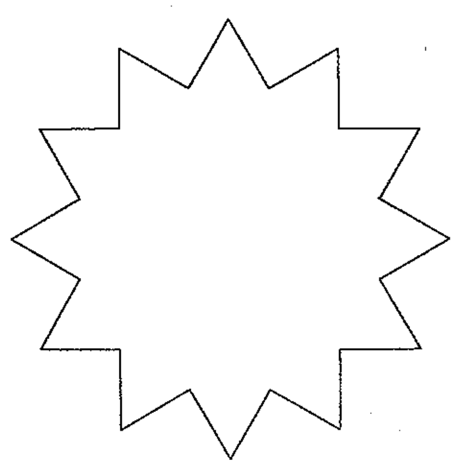
13. The two quadrilaterals are similar. Find the values of x and y.



14. Prove: If $\Delta ABC \sim \Delta PQR$ and $\Delta PQR \sim \Delta XYZ$, then $\Delta ABC \sim \Delta XYZ$.

15. Prove: If $\Delta ABC \cong \Delta PQR$, then $\Delta ABC \sim \Delta PQR$. What is the ratio of similarity in that case?

16. On the facing (heavy-duty) page is a star with interior lines forming 24 pieces. Cut out the pieces carefully, and rearrange them to form three new stars, each similar to the original. [The shape of the star is shown to the right, so that after you have cut the pieces out, you know what the results are to look like.]



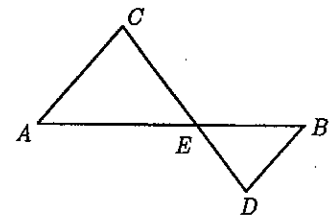
5.3 Exercises

1. In ΔFUN and ΔWHA, $\angle N \cong \angle W$ and $\angle U \cong \angle A$. Draw an appropriate figure and identify the similar triangles carefully.

2. If $\Delta DTB \sim \Delta TDB$, then why is ΔDTB isosceles?

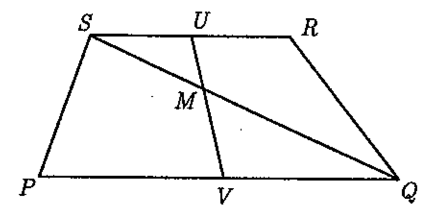
3. In trapezoid $ABCD$, the parallel bases are $\overline{AB}$ and $\overline{CD}$. The diagonals intersect at K.
   a. Which is correct: $\Delta ABK \sim \Delta CDK$ or $\Delta ABK \sim \Delta DCK$?
   b. If $AB = 8$ and $CD = 6$, then what is the scale factor for the similarity?
   c. If $AB = 8$, $CD = 6$, and $BD = 7$, then what is the length of $\overline{BK}$?

4. Given the figure with $\overline{AC} \parallel \overline{BD}$.
Prove that:
a. $\Delta ACE \sim \Delta BDE$
b. $\frac{AE}{BE} = \frac{CE}{DE}$

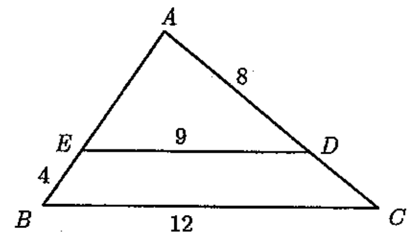


5. Two angles of ΔABC have measures of 15 and 45. Two angles of ΔPQR have measures of 45 and 120. Are the triangles similar?

6. In trapezoid $PQRS$, $\overline{SR} \parallel \overline{PQ}$ and U and V are midpoints.
Prove that $US \cdot MQ = VQ \cdot MS$.



7. Given the figure with $\overline{ED} \parallel \overline{BC}$.
Find the lengths AC, AE, and AB.

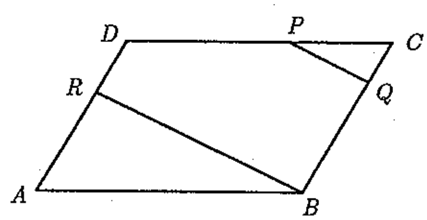


8. Prove: In two similar triangles, corresponding altitudes are in the same ratio as corresponding sides.

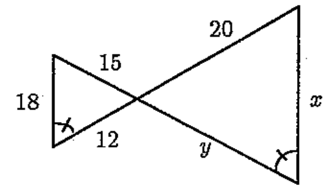
9. Prove: In two similar triangles, corresponding angle bisectors are in the same ratio as corresponding sides.

10. Given: Parallelogram $ABCD$; $\overline{PQ} \parallel \overline{RB}$.

Prove: $AB \cdot CQ = AR \cdot CP$

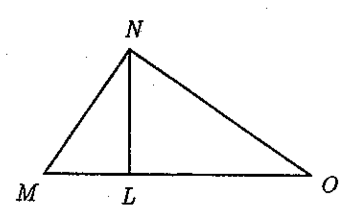


11. Find the values of x and y.



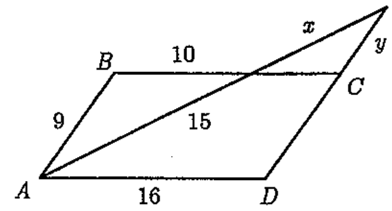
12. Given: $\overline{MN} \perp \overline{NO}$ and $\overline{NL} \perp \overline{MO}$

Prove: $MN \cdot LN = ML \cdot NO$



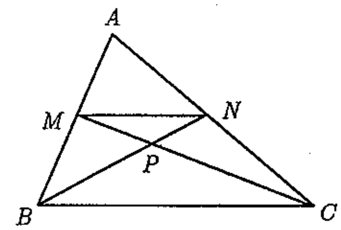
13. Quadrilateral $ABCD$ is a parallelogram.

Find the values of x and y.



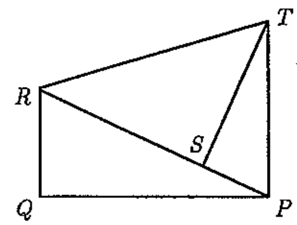
14. Given: $\overline{CM}$ and $\overline{BN}$ are medians of ΔABC.

Prove: $\frac{BP}{PN} = \frac{2}{1}$ and $\frac{CP}{PM} = \frac{2}{1}$.

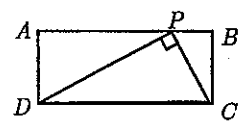


15. In the figure, $\overline{RQ} \perp \overline{PQ}$, $\overline{PT} \perp \overline{PQ}$, and $\overline{ST} \perp \overline{PR}$.

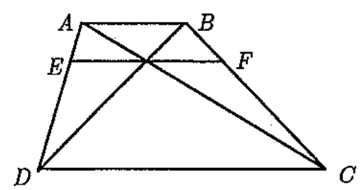
Prove that $ST \cdot RQ = PS \cdot PQ$.



16. Right ΔDPC lies inside rectangle $ABCD$ as shown. Prove: $PD^2 = AP \cdot DC$.

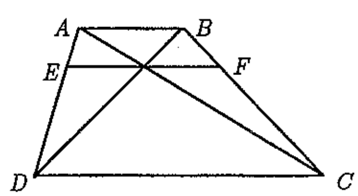


17. $ABCD$ is a trapezoid with $\overline{EF} \parallel \overline{CD}$;
$AB = 8$ and $CD = 24$.
Find the length of $\overline{EF}$.



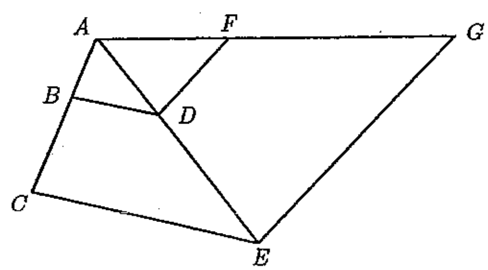
18. $ABCD$ is a trapezoid with $\overline{EF} \parallel \overline{CD}$, where $\overline{EF}$ passes through the intersection point K of the diagonals. Prove that K is the midpoint of $\overline{EF}$.

19. $ABCD$ is a trapezoid with $\overline{EF} \parallel \overline{CD}$;
$AB = a$ and $CD = b$.
Find the length of $\overline{EF}$ in terms of a and b.

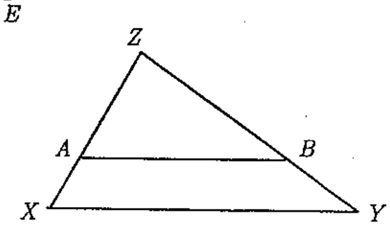


5.4 Exercises

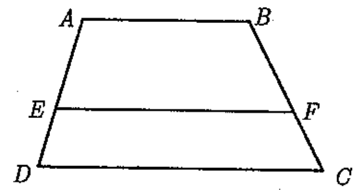
1. In the figure, $\overline{BD} \parallel \overline{CE}$, $\overline{DF} \parallel \overline{EG}$, $AB = 2$, $AC = 5$, $DE = 6$, and $FG = \frac{16}{3}$. Find the length of $\overline{AF}$.



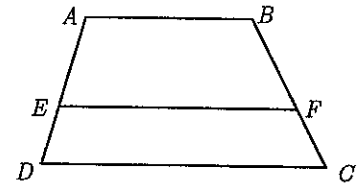
2. $\overline{AB} \parallel \overline{XY}$ in ΔXYZ.
   a. If $AZ = 12$, $AB = 14$, and $AX = 3$, what is XY?
   b. If $AZ = 9$, $XZ = 15$, and $BY = 5$, what is BZ?
   c. If $XY = AB + 3$, $YZ = AB$, and $ZB = 4$, what is AB?



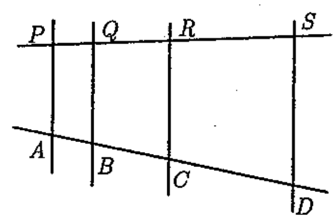
3. $ABCD$ is a trapezoid with $\overline{EF} \parallel \overline{CD}$; $AB = 8$, $CD = 24$, $AE = 9$, $ED = 3$, and $BC = 16$.
   a. Find BF and FC.
   b. Find EF.
   c. Find (perimeter of $ABFE$) : (perimeter of $EFCD$).



4. $ABCD$ is a trapezoid with $\overline{EF} \parallel \overline{CD}$; $AB = 10$, $CD = 30$, and $AE : ED = 4 : 1$.
   a. Find EF.
   b. If we add the condition that (perimeter of $ABFE$) = (perimeter of $EFCD$), then find the perimeter of $ABCD$.

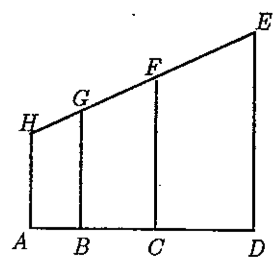


5. In the figure, $\overline{AP} \parallel \overline{BQ} \parallel \overline{CR} \parallel \overline{DS}$ and $AB : BC : CD = 1 : 2 : 3$. If $PR = 12$, find PQ and RS.

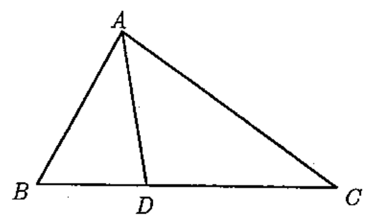


6. The vertical lines are all parallel with $AB = 20$, $BC = 30$, and $CD = 40$.

   a. If $HE = 160$, find the lengths of $\overline{HG}$, $\overline{GF}$, and $\overline{FE}$.
   b. For how many lengths of $\overline{HE}$ between 100 and 200 will all three of the lengths HG, GF, and FE be integers?



7. In ΔABC, $\overline{AD}$ bisects $\angle BAC$; $AB = 8$, $AC = 10$, and $BC = 12$. Find:
   a. $BD : DC$ b. BD



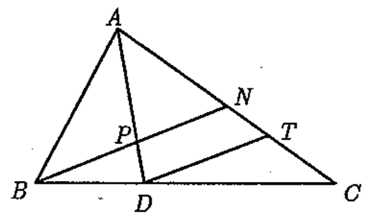
8. The sides of a triangle have lengths 7, 9, and 12. Find the lengths of the two segments cut off on the longest side by the bisector of the largest angle.

9. In ΔABC, $BC = 12$, $AC = 13$, and $AB = 14$. If M is the midpoint of $\overline{AC}$ and P is the point where $\overline{AC}$ is intersected by the bisector of $\angle B$, find the length of $\overline{MP}$.

10. In ΔABC, $\overline{AD}$ bisects $\angle BAC$; N is the midpoint of $\overline{AC}$ and $\overline{DT} \parallel \overline{BN}$. $AB = 72$, $AC = 90$, and $BC = 108$. Find:

a. BD b. $CT : TN$

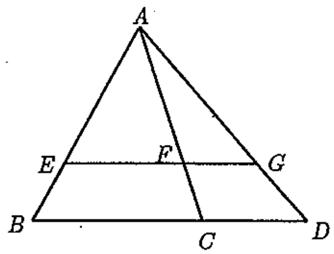
c. NT d. $\frac{AP}{PD}$

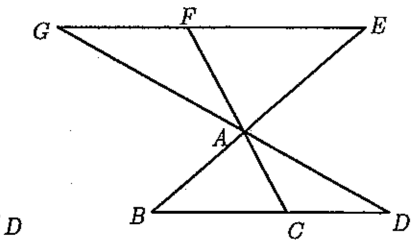


11. Prove indirectly that it is impossible for the trisectors of an angle to trisect the side opposite that angle.

12. Given: In each of the diagrams to the right, $\overline{EG} \parallel \overline{BD}$.

Prove: $\frac{EF}{BC} = \frac{FG}{CD}$

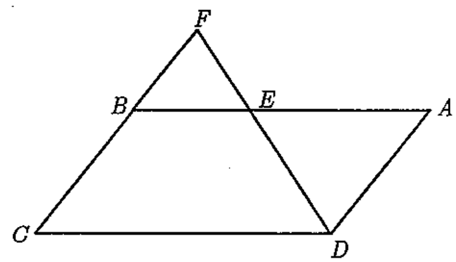




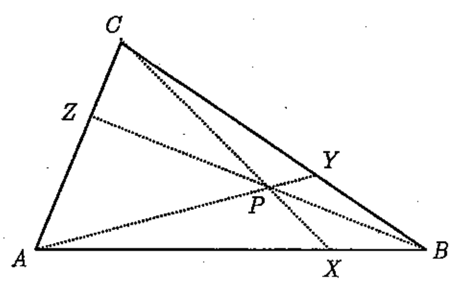
13. Prove indirectly: If X and Y are both on $\overline{AB}$ and if $\frac{AX}{XB} = \frac{AY}{YB}$, then X and Y are, in fact, the same point.

14. Given: $ABCD$ is a parallelogram. $\overline{CB}$ is extended to an arbitrary point F and $\overline{DF}$ is drawn.

Prove: $\frac{AE}{ED} \cdot \frac{DF}{CD} = 1$



15. If P is any point in the interior of ΔABC, then prove that $\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA} = 1$. [Hint: Draw the line through C that is parallel to $\overline{AB}$ and extend both $\overline{AY}$ and $\overline{BZ}$ to meet that parallel. Set up proportions for $\frac{AX}{XB}$, $\frac{BY}{YC}$, and $\frac{CZ}{ZA}$.]

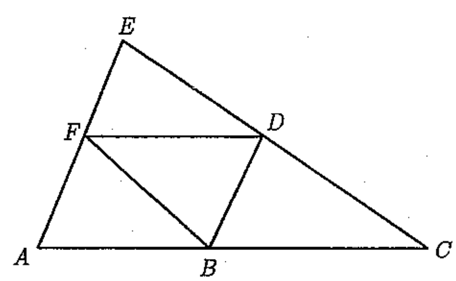


16. Given: $\overline{FD} \parallel \overline{AC}$

$\overline{BD} \parallel \overline{AE}$

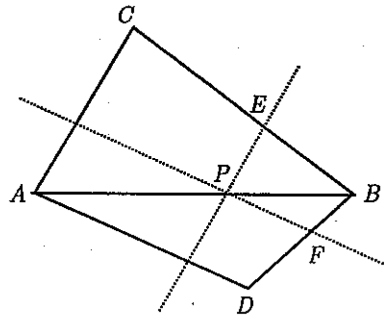
$\overline{FB} \parallel \overline{EC}$

Prove: B, D, and F are the midpoints of $\overline{AC}$, $\overline{CE}$, and $\overline{AE}$.

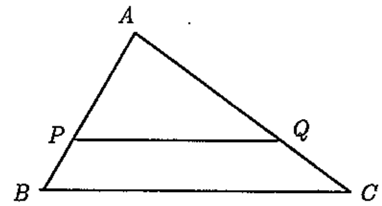


17. On the common base $\overline{AB}$ of ΔABC and ΔABD a point P is chosen. Through P draw a line that is parallel to $\overline{AC}$ and that intersects $\overline{BC}$ at E. Also through P, draw a line parallel to $\overline{AD}$ intersecting $\overline{BD}$ at F. Prove that $\overline{EF} \parallel \overline{CD}$.

(Hint: Find $\frac{AP}{PB}$ two different ways.)



18. Prove indirectly: If $\frac{AP}{PB} = \frac{AQ}{QC}$, then $\overline{PQ} \parallel \overline{BC}$.

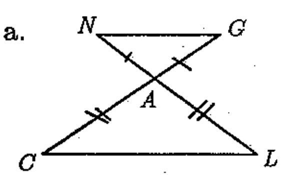


19. Point D is the midpoint of side $\overline{AB}$ in ΔABC. The bisectors of $\angle CDA$ and $\angle CDB$ intersect $\overline{AC}$ and $\overline{BC}$ at points E and F, respectively. Prove that $\overline{EF} \parallel \overline{AB}$.

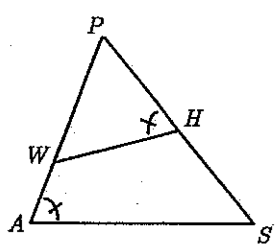
5.5 Exercises

1. In each of the following, identify whether or not there are similar triangles. If there are, set up the similarity and specify the reason (AA, SAS, or SSS) to justify it.

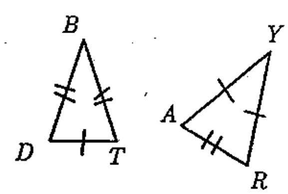
a.



b.

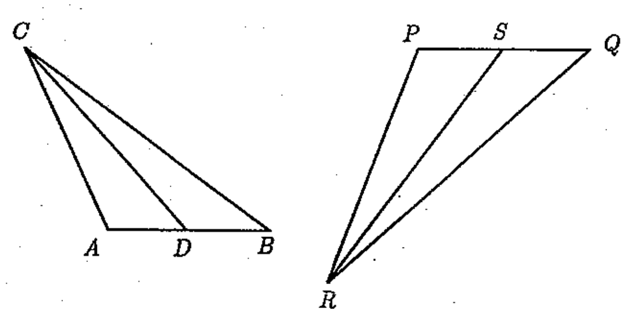


c.



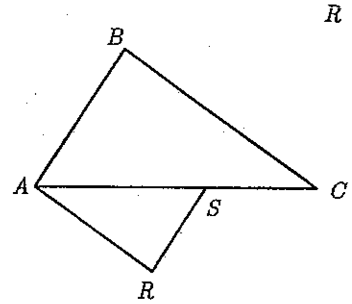
2. Given: $\Delta ADC \sim \Delta PSR$.
$\overline{CD}$ and $\overline{RS}$ are medians.

Prove: $\Delta ABC \sim \Delta PQR$

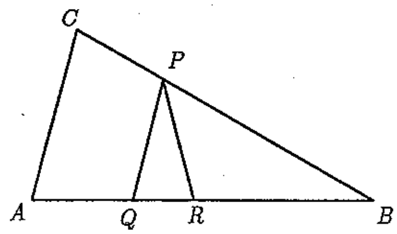


3. Given: $\frac{AB}{SR} = \frac{BC}{RA} = \frac{CA}{AS}$

Prove: $\overline{BC} \parallel \overline{AR}$



4. In the figure, $\overline{PQ} \cong \overline{PR}$ and $\overline{PQ} \parallel \overline{AC}$. Which of the following statements are true?



a. $\frac{BP}{BC} = \frac{PQ}{AC}$

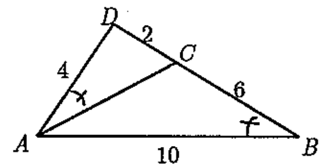
b. $\frac{BP}{BC} = \frac{PR}{AC}$

c. $\Delta BPR \sim \Delta BQP$

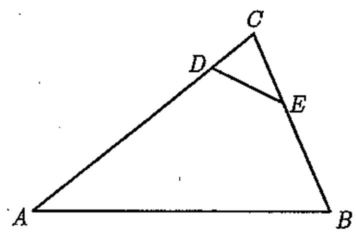
d. $\frac{BP}{BC} = \frac{PQ}{AC}$, $\angle PBQ \cong \angle CBA$, and $\Delta PBQ \sim \Delta CBA$

e. $\frac{BP}{BC} = \frac{PQ}{AC}$, $\angle PBQ \cong \angle CBA$, and $\Delta PBR \sim \Delta CBA$

5. In the figure as marked, determine the similar triangles and then find the length of $\overline{AC}$.



6. In the figure to the right, $AB = 20$, $BE = 7$, $EC = 3$, $CD = 2$, and $DA = 13$. Determine DE.

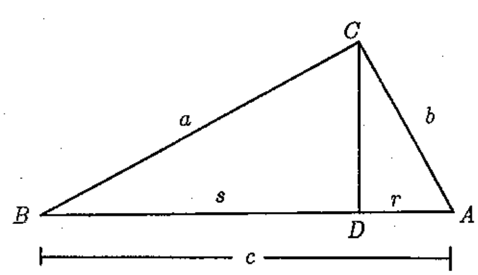


7. Prove that if the vertex angle of one isosceles triangle is congruent to the vertex angle of another isosceles triangle, then the two triangles are similar.

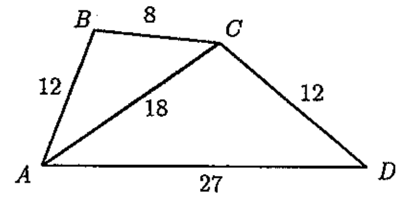
8. ΔABC is isosceles with base $\overline{BC}$ and $\overline{AB} \cong \overline{AC}$. The perpendicular bisector of $\overline{AB}$ meets the line containing the base $\overline{BC}$ at point D. Prove that a leg of the triangle is the geometric mean between the base BC and BD.

9. a. Find lengths for the sides of two triangles so that the triangles are similar, non-congruent, and two sides of one triangle have the same length as two sides of the other triangle.
   b. Can you do this with all sides being integers?

10. In the figure, $\angle ACB$ is a right angle and $\overline{CD}$ is the altitude to the hypotenuse.
   a. Prove that $\Delta ADC \sim \Delta CDB \sim \Delta ACB$.
   b. Prove that $a = \sqrt{cs}$ and $b = \sqrt{cr}$.
   c. Find the value of $a^2 + b^2$.



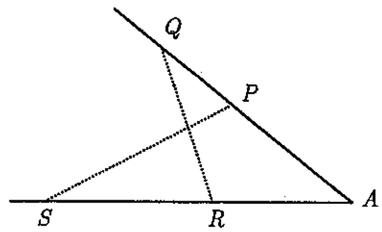
11. In the figure as marked, prove that $\overline{AC}$ bisects $\angle DAB$.



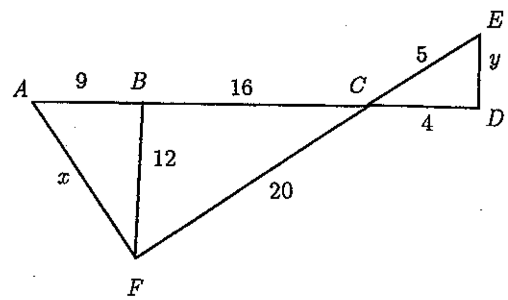
12. How tall is a communication tower that casts a 90-foot shadow at the same time that a 5-foot gatepost casts a 3-foot shadow?

13. Prove that if two triangles are similar, then corresponding medians are in the same ratio as corresponding sides.

14. The points P, Q, R, and S are on the sides of $\angle A$ and $AP \cdot AQ = AR \cdot AS$. Prove that $\Delta APS \sim \Delta ARQ$.



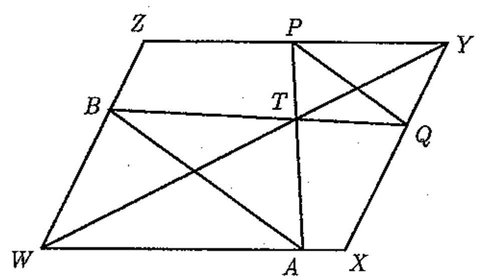
15. In the figure to the right, $\overline{AF} \perp \overline{FE}$ and $\overline{FB} \perp \overline{AD}$. The various segments have the lengths indicated.
   a. Why is $\Delta ABF \sim \Delta FBC$?
   b. Why is $\Delta EDC \sim \Delta FBC$?
   c. Determine x and y.



17. Given: $WXYZ$ is a parallelogram, and T is on diagonal $\overline{WY}$.

Prove: $\Delta ATB \sim \Delta PTQ$

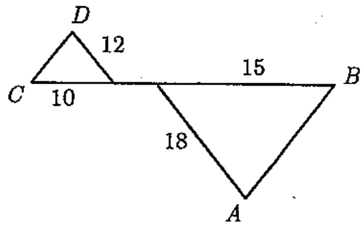
(Hint: Show that both $\frac{AT}{PT}$ and $\frac{BT}{QT}$ are equal to $\frac{TW}{TY}$.)



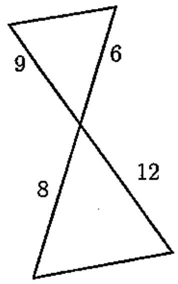
Chapter 5 Review Exercises

1. State whether or not you can conclude that the figure contains two similar triangles. If so, determine the scale factor.

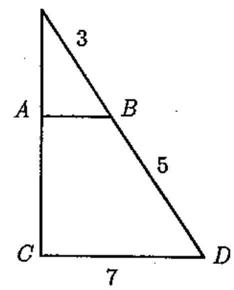
   a. Given $\overline{AB} \parallel \overline{CD}$



   b.

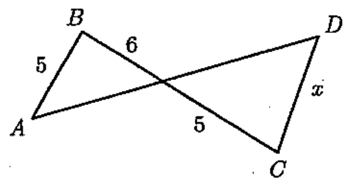


   c. Given $\overline{AB} \parallel \overline{CD}$

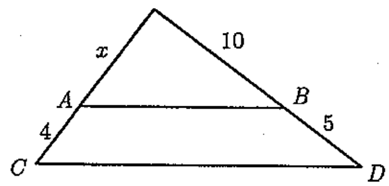


2. Find the value of x in each of the following.

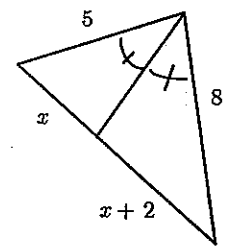
   a. Given $\overline{AB} \parallel \overline{CD}$



   b. Given $\overline{AB} \parallel \overline{CD}$



   c.



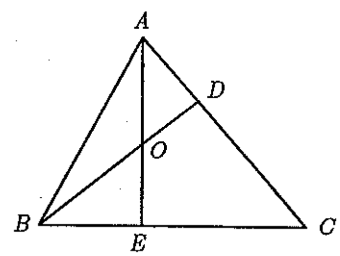
3. a. If $\overline{PQ}$ is 10 inches long, divide $\overline{PQ}$ in the ratio $2:3$.

   b. If $\overline{PQ}$ is 18 units long, divide $\overline{PQ}$ in the ratio $3:2$.

4. The following refer to the figure to the right.

   a. If $\overline{AE} \perp \overline{BC}$ and $\overline{BD} \perp \overline{AC}$, determine all the congruent angles.

   b. If $\angle DBC \cong \angle EAC$, list all the congruent angles.

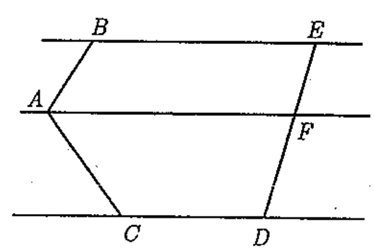


5. Prove that a line drawn through the intersection of the diagonals of a trapezoid and bisecting one of the parallel sides must bisect the other parallel side.

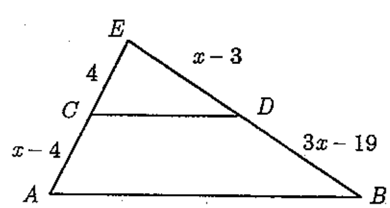
6. Given: $\overleftrightarrow{BE} \parallel \overleftrightarrow{AF} \parallel \overleftrightarrow{CD}$ and $\overrightarrow{AF}$ bisects $\angle BAC$.

   Prove: $\frac{AB}{AC} = \frac{FE}{FD}$

   [Hint: Introduce $\overline{BC}$.]



7. In the figure, $\overline{AB} \parallel \overline{CD}$. Determine EB.

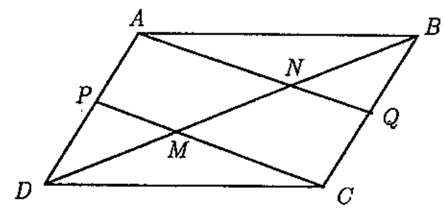


8. In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$. The nonparallel sides $\overline{AD}$ and $\overline{BC}$ intersect at P. Prove that the line through P and the midpoint M of $\overline{CD}$ also passes through the midpoint N of $\overline{AB}$.

9. Prove that in any trapezoid, the diagonals divide each other into segments whose lengths are in the same ratio as the parallel sides of the trapezoid.

10. Given: $ABCD$ is a parallelogram; P and Q are midpoints of $\overline{AD}$ and $\overline{BC}$.

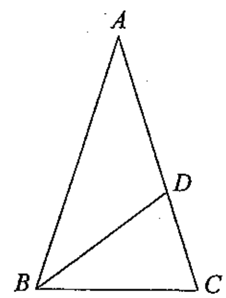
Prove: $DM = MN = NB$



11. Shown is an isosceles triangle in which $AB = AC = 1$ and $BC = x$. The measure of each of the base angles is twice the measure of the vertex angle. $\overline{BD}$ bisects $\angle ABC$.

a. Determine the measure of each angle in the figure.

b. Find the length of $\overline{BC}$ (that is, determine x).

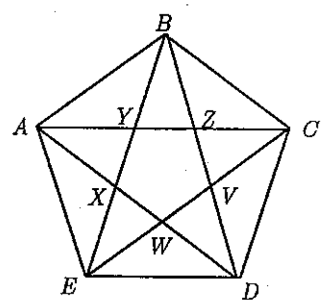


12. $ABCDE$ is a regular pentagon. When the diagonals are drawn to form a five-pointed star, a new regular pentagon is formed inside. Let the side of $ABCDE$ be 1.

a. Find $m\angle YBZ$, $m\angle BED$, and $m\angle VED$.

b. Determine EX, XA, and XY. [Problem 11 will help with this.]

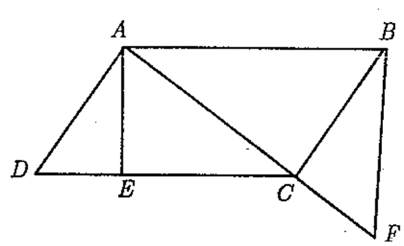
c. What is the ratio of similarity between ΔAXY and ΔADC?



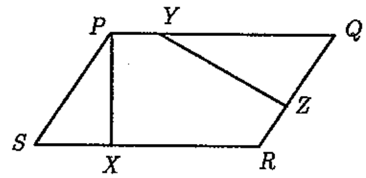
13. The sides of a triangle have lengths 9, 15, and 18. Find the lengths of the two segments into which the longest side is divided by the bisector of the opposite angle.

14. Given: $ABCD$ is a parallelogram; $\overline{BF} \parallel \overline{AE}$.

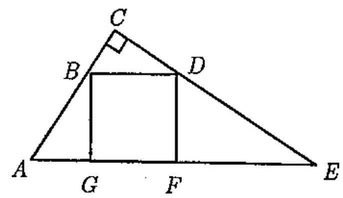
Prove: $\frac{CE}{CD} = \frac{AC}{AF}$



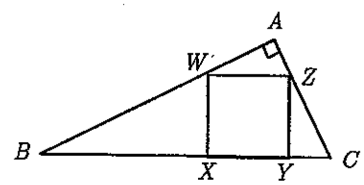
15. Given: $PQRS$ is a parallelogram; $\overline{PX} \perp \overline{RS}$ and $\overline{YZ} \perp \overline{QR}$.
Prove: $PS \cdot QZ = SX \cdot QY$



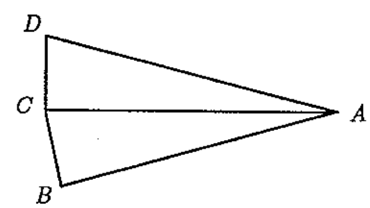
16. Square $BDFG$ is inscribed in right ΔACE with $BD = 5$. Find the value of $AG \cdot FE$.



17. Square $WXYZ$ is inscribed in right ΔABC so that $\overline{XY}$ lies on hypotenuse $\overline{BC}$. Prove that $(XY)^2 = BX \cdot YC$.

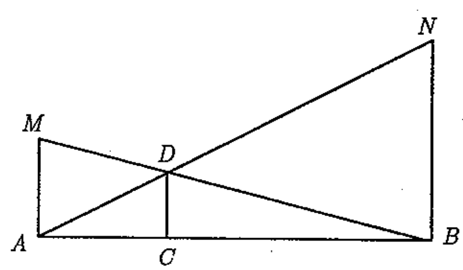


18. Given: $\overrightarrow{AC}$ bisects $\angle BAD$; $\overline{CB} \perp \overline{AB}$ and $\overline{CD} \perp \overline{AC}$. $AB = 7$ and $AC = 8$.
Find: AD

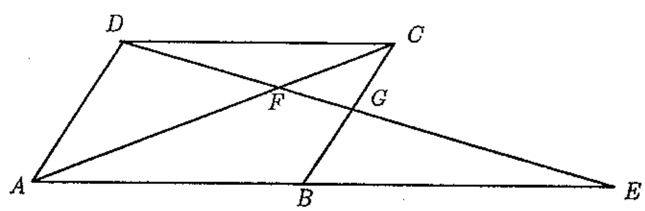


19. In trapezoid $PQRS$, a line is drawn parallel to the bases $\overline{PQ}$ and $\overline{RS}$, intersecting the sides and diagonals at A, B, C, and D, in order. Prove that $AB = CD$.

20. Given: $\overline{AM}$, $\overline{CD}$, and $\overline{BN}$ are all perpendicular to $\overline{AB}$.
Prove: $\frac{1}{AM} + \frac{1}{BN} = \frac{1}{CD}$

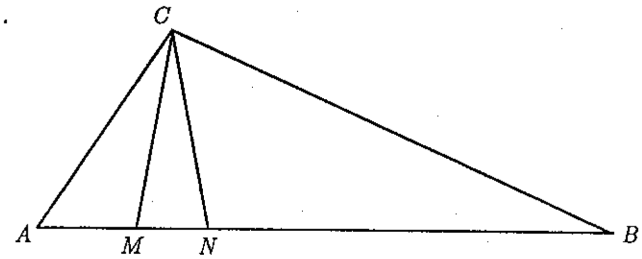


21. Given: $ABCD$ is a parallelogram.
Prove: $(DF)^2 = FG \cdot FE$



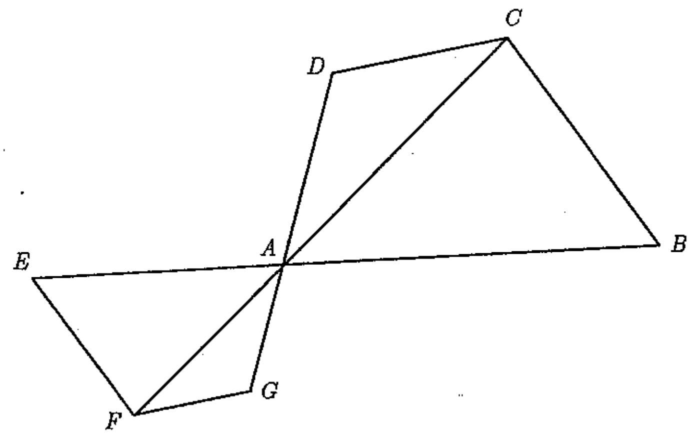
22. <u>Given</u>: $\angle ACM \cong \angle B$ and $CM = CN$.

<u>Prove</u>: $\frac{CN}{BC} = \frac{AM}{AC}$



23. <u>Given</u>: $\frac{AB}{AE} = \frac{AC}{AF} = \frac{AD}{AG}$

<u>Prove</u>: $AGFE \sim ABCD$



24. C is any point on side $\overline{AD}$ of ΔADE. $\overline{CF} \parallel \overline{AE}$ while $\overline{AF} \parallel \overline{ED}$, and $\overline{CF}$ and $\overline{AF}$ intersect at point F outside ΔADE. $\overline{FE}$ intersects $\overline{AD}$ at B. Prove that $(AB)^2 = BC \cdot BD$.

25. $ABCD$ is a parallelogram, and E is a point on $\overrightarrow{AB}$.

Prove that $\frac{FE}{DF} - \frac{BE}{DC} = 1$.