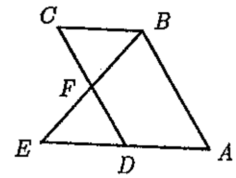
Exercises 4.1

1. Prove **Theorem 4.2**: If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent.

2. Prove **Theorem 4.4**: Consecutive angles of a parallelogram are supplementary.

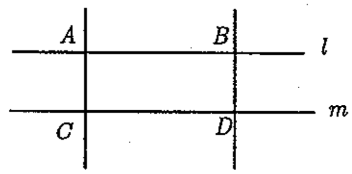
3. Given: $ABCD$ is a parallelogram. D is the midpoint of $\overline{EA}$.

Prove: F is the midpoint of $\overline{BE}$.



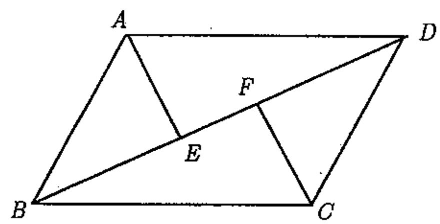
4. Given: $l \parallel m$, $\overline{AC} \perp m$, and $\overline{BD} \perp m$.

Prove: $\overline{AC} \cong \overline{BD}$



5. Given: $ABCD$ is a parallelogram.
$\overline{AE} \perp \overline{BD}$
$\overline{CF} \perp \overline{BD}$

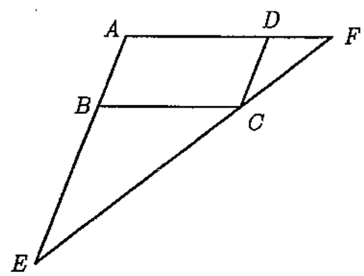
Prove: $\overline{AE} \cong \overline{CF}$



6. If $ABCD$ is a parallelogram and $\overline{AE}$ is perpendicular to $\overline{CD}$ at E and $\overline{CF}$ is perpendicular to $\overline{AB}$ at F, then prove that $DE = BF$.

7. Given: $ABCD$ is a parallelogram.
$BE = BC$
$DF = DC$

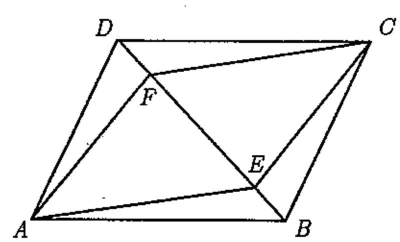
Prove: $\angle E \cong \angle F$
[Use a paragraph proof and let $m\angle E = x$.]



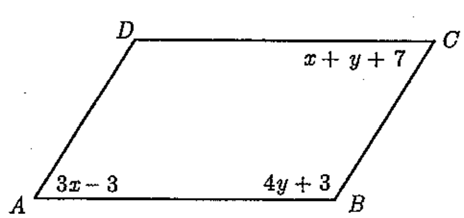
8. $ABCD$ is a parallelogram. Through B a line parallel to $\overline{AC}$ is drawn intersecting $\overleftrightarrow{AD}$ at E. Prove that $\overline{AE} \cong \overline{AD}$.

9. Given: $ABCD$ is a parallelogram.
$\overline{DF} \cong \overline{BE}$

Prove: $\Delta ABE \cong \Delta CDF$

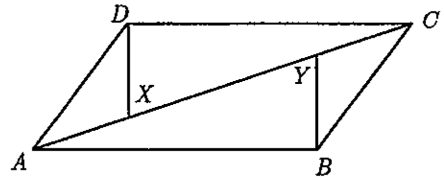


10. In the figure to the right, $ABCD$ is a parallelogram. Determine x and y.



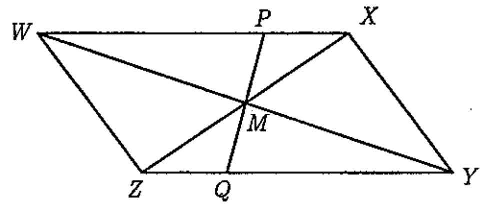
11. Given: $ABCD$ is a parallelogram.
$\overline{AX} \cong \overline{CY}$

Prove: $\angle AXD \cong \angle CYB$



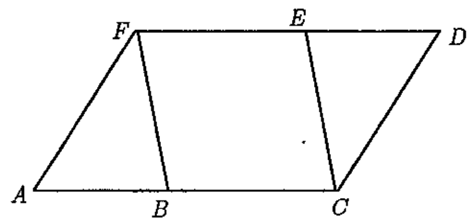
12. Given: $WXYZ$ is a parallelogram.

Prove: $\overline{PX} \cong \overline{QZ}$.



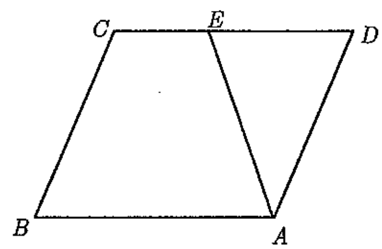
13. Given: $ACDF$ is a parallelogram.
$\overline{BF} \parallel \overline{CE}$

Prove: $\angle AFB \cong \angle DCE$



14. Given: $ABCD$ is a parallelogram with an acute angle at vertex D.
$AE = AD$.

Prove: $\frac{1}{2} m\angle DAE = 90 - m\angle B$



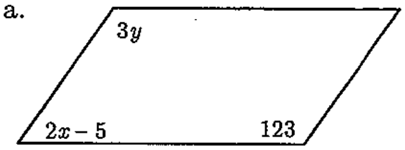
15. In parallelogram $ABCD$, E and F are the midpoints of $\overline{AB}$ and $\overline{CD}$, respectively. $\overline{EC}$ and $\overline{BF}$ intersect at G, while $\overline{ED}$ and $\overline{AF}$ intersect at H. Prove that $\overline{EF}$ and $\overline{GH}$ bisect each other.

Exercises 4.2

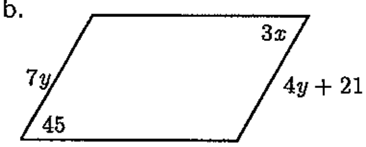
1. Prove **Theorem 4.8**: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

2. What values of x and y make each of the quadrilaterals a parallelogram?

a.



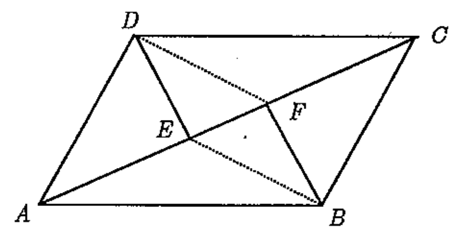
b.



3. In ΔABC, the median $\overline{BP}$ is extended through P to R with $BP = PR$, and the median $\overline{CQ}$ is extended through Q to S with $CQ = QS$. Prove that A, R, and S are collinear. (Hint: Show that $\overline{AR}$ and $\overline{AS}$ are each parallel to the same line and then apply the Parallel Postulate.)

4. Given: $ABCD$ is a parallelogram.
$\overline{DE} \perp \overline{AC}$
$\overline{BF} \perp \overline{AC}$

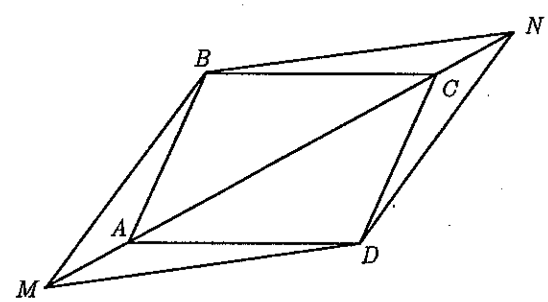
Prove: $DEBF$ is a parallelogram.



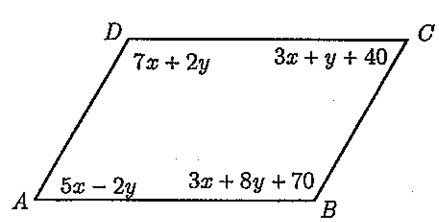
5. Given: $ABCD$ is a parallelogram. M is on $\overline{AB}$ and N is on $\overline{CD}$ such that $AM = CN$.

Prove: $MBND$ is a parallelogram.

6. Diagonal $\overline{AC}$ of parallelogram $ABCD$ is extended to points M and N such that M-A-C-N. If $AM = CN$, then prove that $MBND$ is a parallelogram.

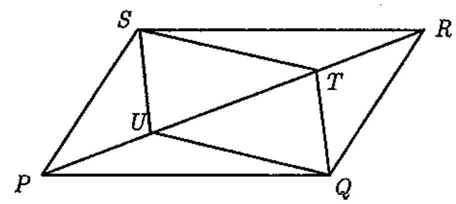


7. Show that $ABCD$ is not a parallelogram (use an indirect proof).

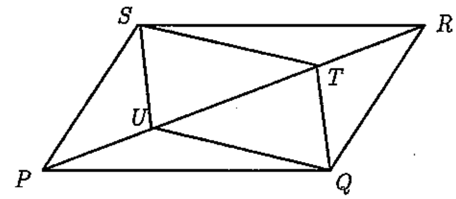


8. Given: $\Delta PUQ \cong \Delta RTS$

Prove: $SUQT$ is a parallelogram.



9. Given: $PQRS$ is a parallelogram.
$\overline{PU} \cong \overline{TR}$.



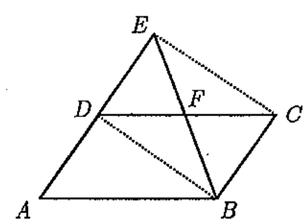
There are several different ways to prove that $SUQT$ is a parallelogram.

Outline such a proof in which the final reason (to justify that $SUQT$ is a parallelogram) is the following.

a. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
b. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
c. If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.
d. If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.

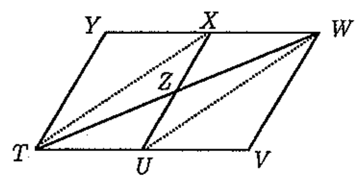
10. Given: $ABCD$ is a parallelogram.
D is the midpoint of $\overline{AE}$.

Prove: F is the midpoint of $\overline{BE}$.



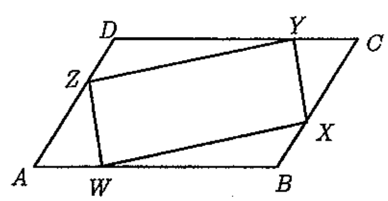
11. Given: $TVWY$ is a parallelogram.
U is the midpoint of $\overline{TV}$.
Z is the midpoint of $\overline{TW}$.

Prove: $\overline{TX} \cong \overline{UW}$



12. Given: $ABCD$ is a parallelogram.
$AW = BX = CY = DZ$.

Prove: $WXYZ$ is a parallelogram.



Exercises 4.3

1. **Prove Theorem 4.10:** If a quadrilateral is a rectangle then the diagonals are congruent.

2. **Prove Theorem 4.11:** If a quadrilateral is a rhombus then the diagonals are perpendicular to each other.

3. **Prove Theorem 4.12:** If a quadrilateral is a rhombus then the diagonals bisect the angles of the rhombus.

4. Prove that the midpoint of the hypotenuse of a right triangle is equidistant from the three vertices. [Hint: Use the right triangle as the start of a rectangle.]

5. Prove that if a point is the midpoint of one side of a triangle and this point is equidistant from the vertices of the triangle, then the triangle is a right triangle and the point must lie on the hypotenuse.

6. Prove that if a single angle of a parallelogram is a right angle then the parallelogram is a rectangle.

7. Prove that if two consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus.

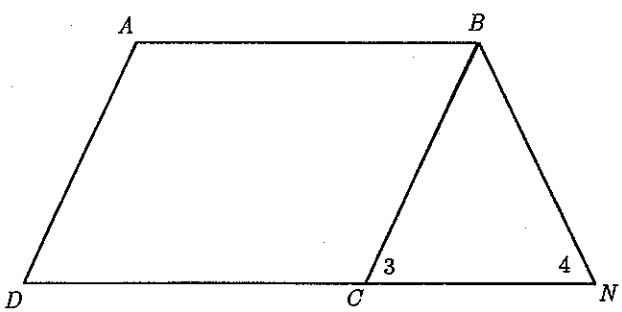
8. Prove that if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

9. Prove that if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

10. Prove that if the diagonals of a parallelogram bisect the angles of the parallelogram, then the parallelogram is a rhombus.

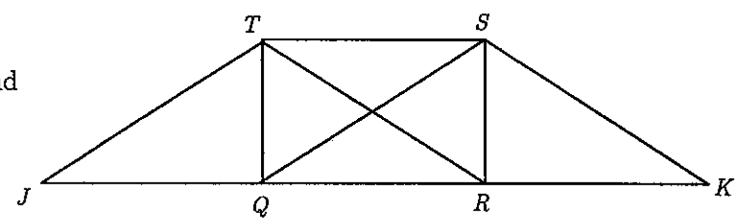
11. Given: $ABCD$ is a parallelogram.
$DC = BN$
$\angle 3 \cong \angle 4$

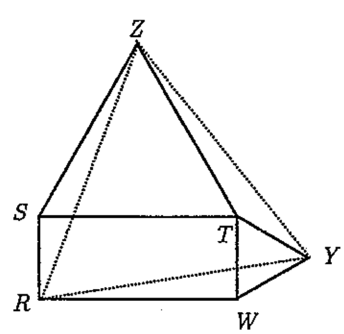
Prove: $ABCD$ is a rhombus.



12. Given: Rectangle $QRST$, parallelogram $RKST$, and parallelogram $JQST$.

Prove: $\overline{TJ} \cong \overline{SK}$



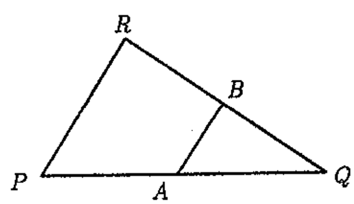


13. In the figure to the right, $RSTW$ is a rectangle, while ΔYWT and ΔSTZ are both equilateral. What can you say about ΔRYZ? Can you prove it?

14. Is a square a rhombus? Explain.

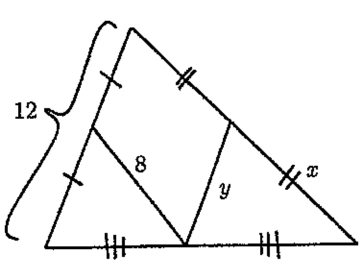
Exercises 4.4

1. In ΔPQR, A and B are midpoints.
   $PR = 16$, $m\angle P = 58$, and $m\angle Q = 38$.
   Find:
   a. AB b. $m\angle ABR$

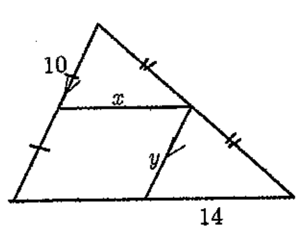


2. Determine the value of x and y in each of the following.

a.



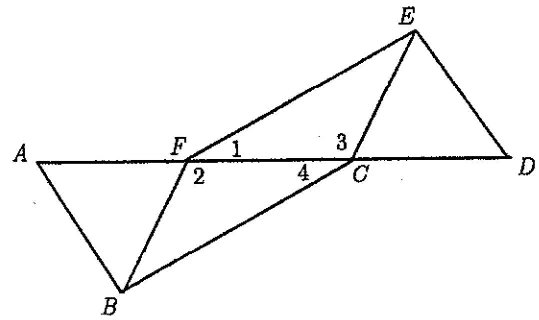
b.



3. Prove that the quadrilateral formed by joining in order the midpoints of the sides of any quadrilateral is always a parallelogram.

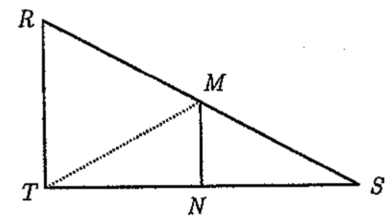
4. <u>Given</u>: $\overline{AB} \parallel \overline{ED}$
$\angle 2 \cong \angle 3$
$\overline{BF}$ is a median in ΔABC.
$\overline{EC}$ is a median in ΔDEF.

<u>Prove</u>: $BCEF$ is a parallelogram.



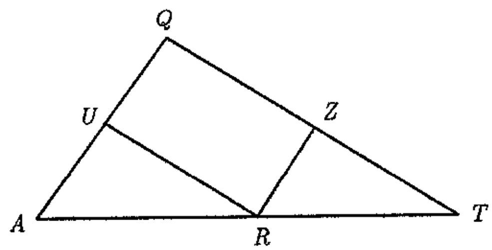
5. <u>Given</u>: $m\angle RTS = 90$
$\overline{MN}$ is the perpendicular bisector of $\overline{TS}$.

<u>Prove</u>: $\overline{TM}$ is a median.



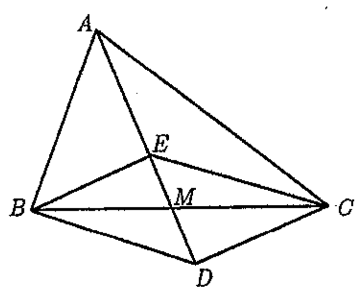
6. <u>Given</u>: U, Z, and R are midpoints of the sides of ΔQAT.

<u>Prove</u>: $\Delta ARU \cong \Delta RTZ$



7. <u>Given</u>: $\overline{AM}$ is a median of ΔABC.
$\overline{BE} \perp \overline{AD}$
$\overline{CD} \perp \overline{AD}$

<u>Prove</u>: $BECD$ is a parallelogram.



8. Prove the **Corollary** to the Midline Theorem <u>indirectly</u>:
If a line passes through the midpoint of one side of a triangle and is parallel to a second side, then it must pass through the midpoint of the third side of the triangle.

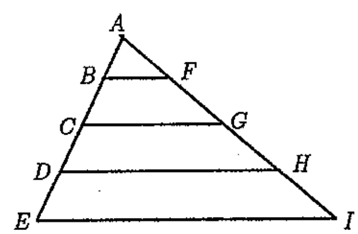
9. A **kite** is a quadrilateral $ABCD$ in which $AB = BC$ and $CD = DA$. [There are two pairs of congruent adjacent sides, but no side is in both pairs.] If P is a point on diagonal $\overline{BD}$ of kite $ABCD$ with the congruent sides identified above, why is PA equal to PC?

10. In ΔABC, let P, Q, and R be the midpoints of sides $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. Without using an area formula, explain why the area of ΔAPR is one-fourth the area of ΔABC.

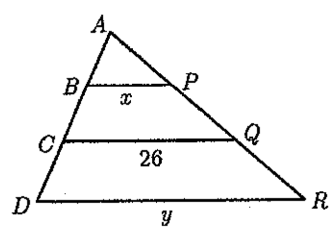
Exercises 4.5

1. The bases of a trapezoid have lengths $2x+7$ and $5x+8$. If the median of the trapezoid has a length of $3x+10$, then find the value of x.

2. In the figure, $AB = BC = CD = DE$ and $AF = FG = GH = HI$. If $EI = 16$, determine the lengths of BF, CG, and DH.

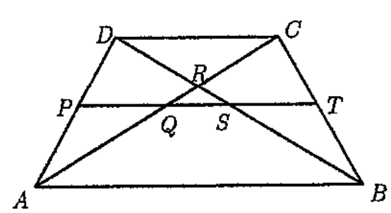


3. Find the values of x and y if $AB = BC = CD$ and $AP = PQ = QR$.



Problems 4–7 refer to the trapezoid pictured to the right.

Given: $ABCD$ is an isosceles trapezoid with bases $\overline{AB}$ and $\overline{CD}$ and median $\overline{PT}$.



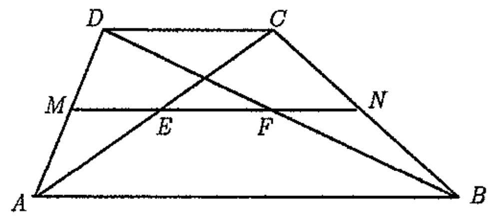
4. Prove: $\overline{AC} \cong \overline{BD}$

5. Prove: $\Delta DAB \cong \Delta CBA$

6. Prove: $\Delta DAR \cong \Delta CBR$

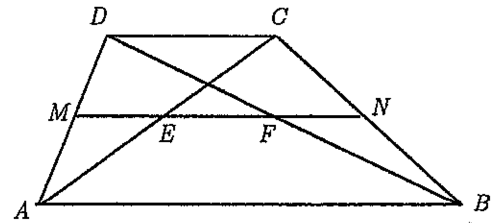
7. What are the isosceles triangles in the figure? Prove them isosceles.

8. $\overline{MN}$ is the median of trapezoid $ABCD$. The bases have lengths $AB = 16$ and $DC = 6$. Determine the lengths ME, EF, and FN.



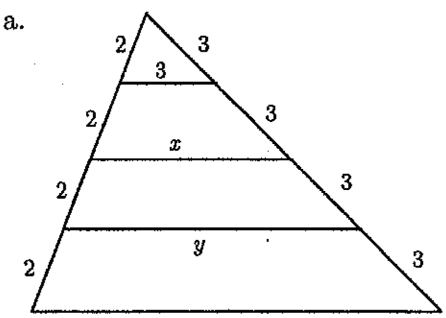
9. Given: $ABCD$ is a trapezoid.
$\overline{MN}$ is the median.
Let $AB = x$ and $CD = y$.

Prove: $EF = \frac{1}{2}(AB - DC)$

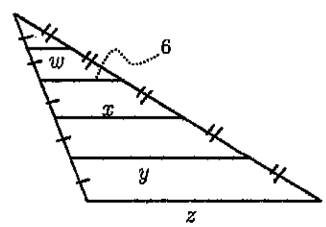


10. Determine the values of the variables in each of the following.

a.



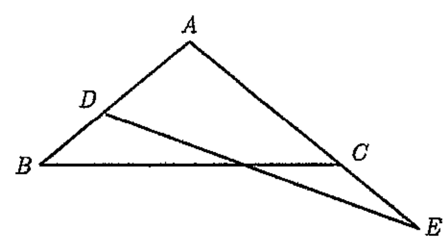
b.



11. Prove: The median of a trapezoid bisects each of the diagonals of the trapezoid.

12. Given: $AB = AC$ and $BD = CE$.

Prove: $\overline{BC}$ bisects $\overline{DE}$.
[Hint: Construct a trapezoid and use problem 11.]



13. Prove that the median of a trapezoid does **not** pass through the intersection point of the diagonals.

14. In a trapezoid, the median is **trisected** by the diagonals. Letting the longer base have length $2x$ and the shorter base have length $2y$, find (algebraically) the relationship between the lengths of the two bases.

Exercises 4.6

1. Draw a scalene triangle all of whose angles are acute. Then sketch the locations of the incenter, the circumcenter, the orthocenter, and the centroid of the triangle.

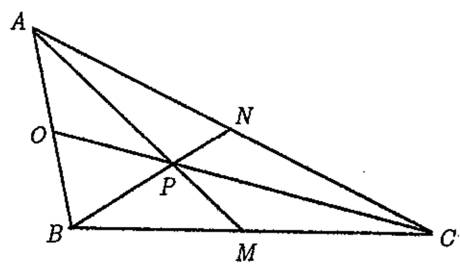
2. Draw a right triangle. Then sketch the locations of the incenter, the circumcenter, the orthocenter, and the centroid of the triangle.

3. Draw a scalene triangle one of whose angles is obtuse. Then sketch the locations of the incenter, the circumcenter, the orthocenter, and the centroid of the triangle.

4. Use your results from problems 1, 2, and 3 to complete the following table telling under what conditions the given points are inside, on, or outside the triangle.

Angles of Δ	Incenter	Circumcenter	Orthocenter	Centroid
All acute	inside			
One right				
One Obtuse			outside	

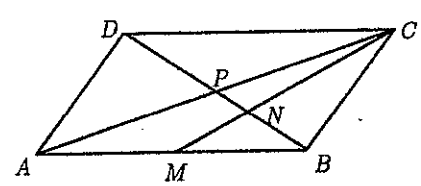
5. In the figure, P is the centroid of ΔABC.
   a. If $AP = 6$, find PM.
   b. If $PN = 5$, find BN.
   c. If $CO = 24$, find PO.
   d. If $PA + PB + PC = 24$, find $AM + BN + CO$.



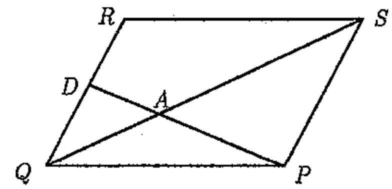
6. Explain why the incenter, circumcenter, orthocenter, and centroid of an equilateral triangle are all the same point.

7. Prove that the incenter, circumcenter, orthocenter, and centroid of an isosceles triangle are collinear.

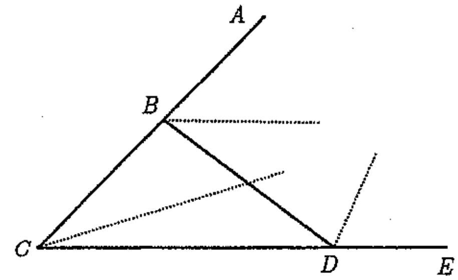
8. In the figure, $ABCD$ is a parallelogram and M is the midpoint of $\overline{AB}$.
   a. Where is P along $\overline{AC}$?
   b. What special name is given to point N?
   c. Where is N along $\overline{BP}$?
   d. Where is N along the diagonal $\overline{BD}$?



9. In parallelogram $PQRS$, D is the midpoint of $\overline{QR}$. If $QA = 8$, determine QS.



10. Given ΔBCD, prove that the bisectors of the exterior angles $\angle ABD$ and $\angle BDE$ and the bisector of the interior $\angle BCD$ are concurrent.

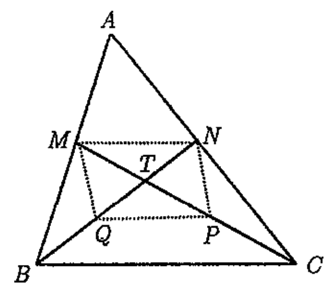


11. Given ΔABC with $\overline{BA} \cong \overline{BC}$. Let the lines drawn perpendicular to $\overline{BA}$ and $\overline{BC}$ at A and C intersect at P. Prove that P lies on the bisector of $\angle ABC$.

12. Given: O is the orthocenter of ΔPQR.

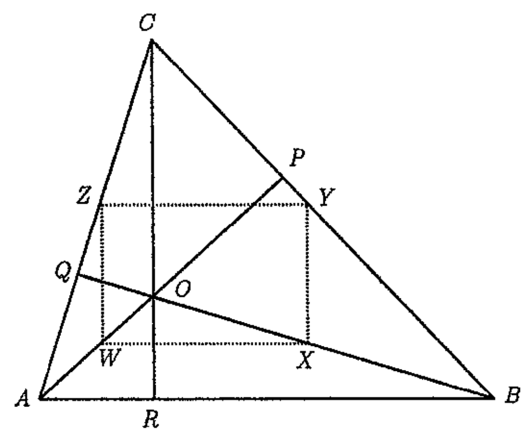
Prove: P is the orthocenter of ΔOQR.

13. In ΔABC, $\overline{BN}$ and $\overline{CM}$ are medians intersecting at T while P and Q are midpoints of $\overline{CT}$ and $\overline{BT}$, respectively. Prove that $MNPQ$ is a parallelogram.



14. Given: In ΔABC, $\overline{AP}$, $\overline{BQ}$, and $\overline{CR}$ are the altitudes, intersecting at O. Points W and X are midpoints of $\overline{AO}$ and $\overline{BO}$, respectively, while Y and Z are midpoints of sides $\overline{BC}$ and $\overline{AC}$, respectively.

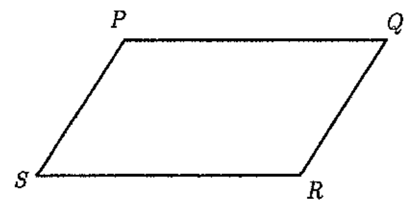
Prove: $WXYZ$ is a rectangle.



15. Adding to the figure of problem 14, let M be the midpoint of $\overline{AB}$ and N the midpoint of $\overline{CO}$.
   a. Why is $WMYN$ a rectangle?
   b. Compare the intersection point of $\overline{WY}$ with $\overline{XZ}$ and that of $\overline{WY}$ with $\overline{MN}$.
   c. Since the diagonals of a rectangle are congruent, what is the consequence of (b)?
   d. Can you do this again with $ZMXN$?

Chapter 4 Review Exercises

1. $PQRS$ is a parallelogram.

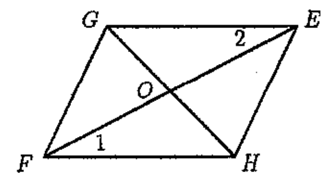


   a. If $PS = SR$, what special kind of parallelogram is $PQRS$?
   b. If $\overline{PS} \perp \overline{SR}$, what special kind of parallelogram is $PQRS$?
   c. If $\overline{PR} \perp \overline{QS}$, what special kind of parallelogram is $PQRS$?
   d. If $PQ = 4x + y$, $QR = y + 4$, $RS = 3x + 6$, and $SP = 2x + y$, determine the lengths of the sides of the parallelogram.

2. a. In parallelogram $ABCD$, if $m\angle CAD = 20$ and $m\angle ACD = 60$, determine the measures of $\angle D$, $\angle B$, $\angle CAB$, and $\angle ACB$.
   b. In parallelogram $ABCD$, if $AB = 2x + y$, $BC = x + 2$, $CD = x + 12$, and $DA = y + 6$, determine x and y.

3. Given: $\angle 1 \cong \angle 2$ and O is the midpoint of $\overline{GH}$.

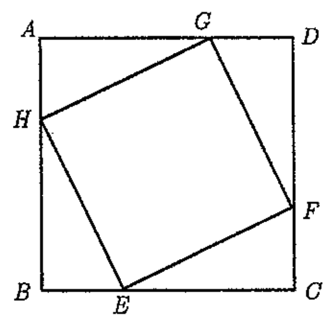
   Prove: $GEHF$ is a parallelogram.



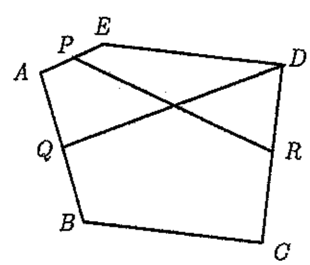
4. Given: $ABCD$ is a square.
   $AH = BE = CF = DG$

   a. Prove: $\Delta AHG \cong \Delta BEH$

   b. Prove: $EFGH$ is a square.

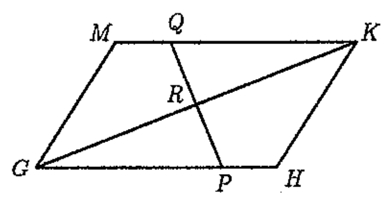


5. Given that $\overline{ED} \parallel \overline{BC}$, $\overline{ED} \cong \overline{BC}$, and points P, Q, and R are midpoints of $\overline{AE}$, $\overline{AB}$, and $\overline{CD}$, respectively, prove that $\overline{QD}$ bisects $\overline{PR}$.
   [Hint: Introduce $\overline{PQ}$ and $\overline{EB}$.]



6. Given: $GHKM$ is a parallelogram and $MQ = HP$.

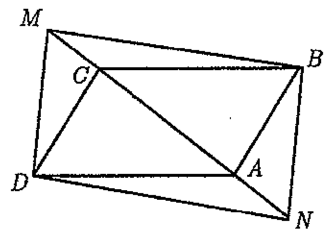
   Prove: $\overline{GK}$ and $\overline{PQ}$ bisect each other.



7. Given a parallelogram $ABCD$ with $AD > AB$. The bisector of $\angle A$ intersects $\overline{BC}$ at G and the bisector of $\angle B$ intersects $\overline{AD}$ at H. Prove that $ABGH$ is a rhombus.

8. <u>Given</u>: $ABCD$ is a parallelogram and $AN = CM$.

<u>Prove</u>: $NBMD$ is a parallelogram.

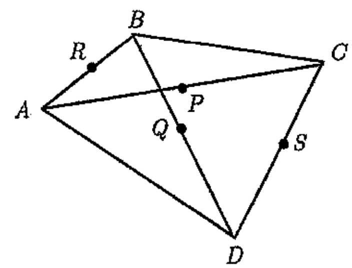


9. Prove that if the diagonals of a trapezoid bisect both base angles at one base, then the trapezoid is isosceles.

10. Prove indirectly: In a scalene triangle, an altitude drawn to one of the sides does not bisect that side.

11. Prove: If the bases of a trapezoid are both perpendicular to one of the legs of the trapezoid, then the segments drawn from the endpoints of that leg to the midpoint of the other leg are congruent.

12. Given quadrilateral $ABCD$ with P, Q, R and S the midpoints of $\overline{AC}$, $\overline{BD}$, $\overline{AB}$, and $\overline{CD}$, respectively. Prove that $PSQR$ is a parallelogram.

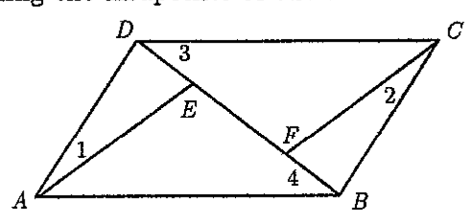


13. Prove: If the midpoints of a rhombus are joined in order, the resulting quadrilateral is a rectangle.

14. Prove that the quadrilateral formed by joining the midpoints of the sides of a kite in order is a rectangle.

15. <u>Given</u>: $AB = CD$ and $BF = DE$.
$\angle 3 \cong \angle 4$

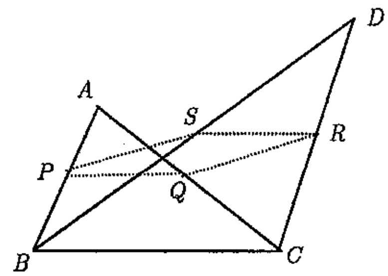
<u>Prove</u>: $\angle 1 \cong \angle 2$



16. The point of intersection of the diagonals of a parallelogram is equidistant from three of the four vertices of the parallelogram. Prove that the parallelogram is a rectangle.

17. Prove: In a trapezoid, the midpoints of the bases and the midpoints of the diagonals are the vertices of a parallelogram.

18. In the figure, P, Q, R, and S are midpoints of $\overline{AB}$, $\overline{AC}$, $\overline{CD}$, and $\overline{BD}$, respectively. Prove that $PQRS$ is a parallelogram.



19. Let $\overline{AB}$ be a diameter of a circle with center O.
   a. Prove that if P is any point of the circle other than A or B, then $\angle APB$ is a right angle.
   b. Prove that if Q is any point in the interior of the circle, but not lying on $\overline{AB}$, then $\angle AQB$ is an obtuse angle.
   c. Prove that if R is any point in the exterior of the circle, but not lying on $\overleftrightarrow{AB}$, then $\angle ARB$ is an acute angle.

20. Let P, Q, and R be three noncollinear points. How would you obtain ΔABC for which P, Q, and R are the midpoints of the sides?

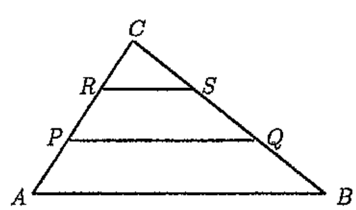
21. In ΔABC, let $\overline{AM}$ and $\overline{BN}$ be the medians from vertices A and B, intersecting at P. Let Q and R be the midpoints of $\overline{AN}$ and $\overline{NC}$, respectively. Draw lines parallel to $\overline{BN}$ through points A, Q, R, and C, and extend $\overline{AM}$ to intersect the last of these parallels.
   a. Why does the parallel through R also pass through M?
   b. How do the five parallel lines divide $\overline{AC}$?
   c. How do the five parallel lines divide the extended median $\overline{AM}$?
   d. Where is point P on median $\overline{AM}$?
   e. If you had used the median from C rather than the one from B, would it have intersected median $\overline{AM}$ in the **same** point P? Why or why not?

22. Lines ℓ, m, and n are three parallel lines that intercept segments $\overline{AB}$ and $\overline{BC}$ on one transversal and segments $\overline{PQ}$ and $\overline{QR}$ on a second transversal. Prove that if $\overline{AB} \not\cong \overline{BC}$, then $\overline{PQ} \not\cong \overline{QR}$.

23. Towns A, B, and C want to build a joint swimming pool located so that it is equidistant from the center of each of the towns. Where should they build the pool?

24. Prove that the segment joining the midpoints of the legs of a right triangle is congruent to the median to the hypotenuse.

25. In ΔABC, $\overline{PQ}$ and $\overline{RS}$ trisect side $\overline{AC}$ and are drawn parallel to $\overline{AB}$. Prove that $PQ + RS = AB$.



Cumulative Review
Number 1

1. For each of the following hypotheses, draw a conclusion and state (or identify) a theorem, corollary, or postulate you would use to justify your conclusion.
   a. In ΔABC, if points M and N are midpoints of $\overline{AB}$ and $\overline{AC}$, respectively, and $MN = 6$, then ... .
   b. In a plane, if $\overline{PQ} \perp \overline{AB}$ and $\overline{AB} \parallel \overline{CD}$, then ... .
   c. In quadrilateral $ABCD$, if M is the midpoint of diagonals $\overline{AC}$ and $\overline{BD}$, then ... .
   d. In ΔXYZ, if $XY = 7$, $YZ = 5$, and $XZ = K$, then K ... .

2. For each of the following statements, specify "True" if the statement is always true. Otherwise specify "False."
   a. If a theorem is true, then its converse is true.
   b. If ΔABC is isosceles with $AB = AC$ and D between B and C, then $AC > AD$.
   c. If $\Delta ABC \cong \Delta ACB$, then ΔABC is isosceles.
   d. If, in right ΔABC, $m\angle B = 90$ and $\overline{BD}$ is a median with $BD = AB$, then $m\angle A = 60$.

3. Which of the following is true for all trapezoids?
   a. Two sides are congruent.
   b. Two angles are congruent.
   c. Two angles are supplementary.
   d. The median joins the midpoints of the bases.

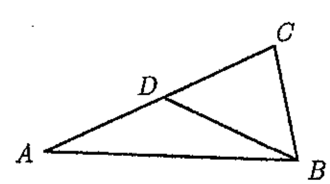
4. If $PQRS$ is a rhombus and $m\angle PQR = 50$, then $m\angle SPR =$.
   a. 60 b. 50 c. 75 d. 65

5. Given $\overline{AB} \cong \overline{CD}$ with point X between A and B and point Y between C and D, determine whether the following implications are true or false.
   a. $AX > XB \rightarrow CY > YD$
   b. $AX > CY \rightarrow XB < YD$
   c. $AX = XB \rightarrow CY = YD$
   d. $AX > YD \rightarrow CY < XB$

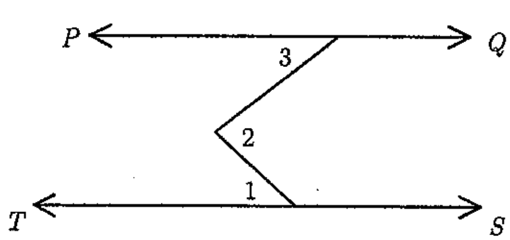
6. Two points A and B are taken on $\overrightarrow{OL}$. On $\overrightarrow{OM}$, two points C and D are taken such that $\overline{OC} \cong \overline{OA}$ and $\overline{OD} \cong \overline{OB}$. $\overline{BC}$ and $\overline{AD}$ intersect at E. Prove that E is on the bisector of $\angle LOM$.

7. An angle has a complement and a supplement, one of which has a measure 6 times the measure of the other. Find the measure of the angle.

8. In the figure to the right, find $m\angle A$ if $AD = DB$, $DC = BC$, and $m\angle C = 76$.



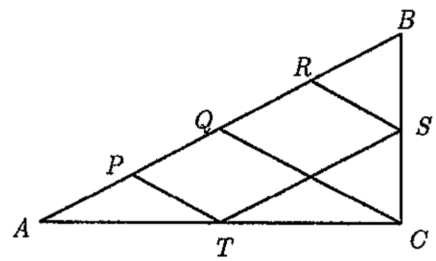
9. In the figure, $\overleftrightarrow{PQ} \parallel \overleftrightarrow{TS}$.
$m\angle 1 = 44$ and $m\angle 2 = 86$.
Find $m\angle 3$.



10. In ΔABC, $\angle C$ is a right angle. Points P, Q, and R divide hypotenuse $\overline{AB}$ into four congruent parts. Points S and T are the midpoints of $\overline{BC}$ and $\overline{AC}$, respectively. $AB = 24$.

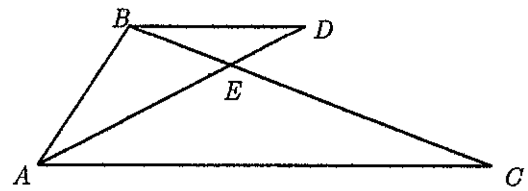
a. Determine QC.

b. Determine the perimeter of $PRST$.



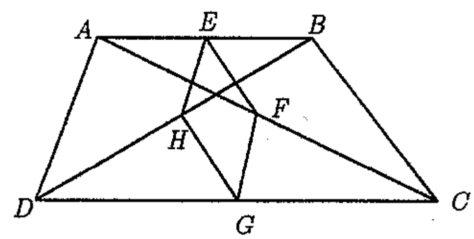
11. Given: ΔABC with $\overrightarrow{AD}$ the bisector of $\angle BAC$ and $\overline{AB} \cong \overline{BD}$.

Prove: $\overline{BD} \parallel \overline{AC}$



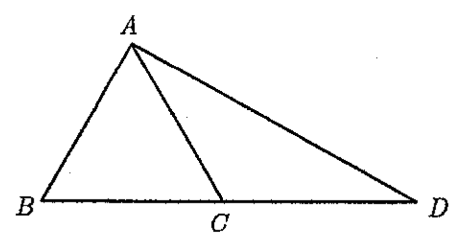
12. Given: $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{CD}$. $\overline{E}$, $\overline{F}$, $\overline{G}$, and H are the midpoints of $\overline{AB}$, $\overline{AC}$, $\overline{CD}$, and $\overline{BD}$, respectively.

Prove: $EFGH$ is a parallelogram.



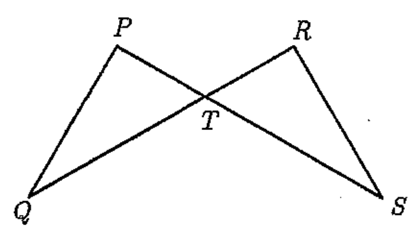
13. Given: ΔABC is equilateral. C is the midpoint of $\overline{BD}$.

Prove: a. $AD > AC$
b. $BD > AD$



14. Given: $\angle P \cong \angle R$; $PS = RQ$; $PQ = RS$.

Prove: $QT = ST$



Cumulative Review
Number 2

1. For each of the following hypotheses, draw a conclusion and state (or identify) a theorem, corollary, or postulate you would use to justify your conclusion.
   a. In ΔABC, if $AC > AB$, then … .
   b If $ABCD$ is a parallelogram and $\overline{AC} \perp \overline{BD}$, then … .
   c. In ΔKLM, if $m\angle L = 90$, R is the midpoint of $\overline{KM}$, and $LR = 4$, then … .
   d. If $\overleftrightarrow{AB} \perp \overline{EF}$ at R with $RE = RF$, and if M is any point other than R on $\overleftrightarrow{AB}$, then … .

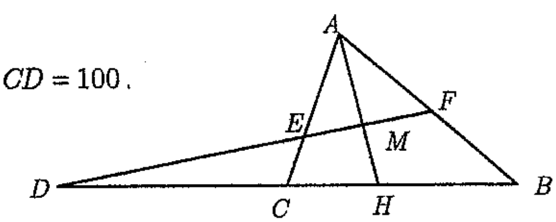
2. For each of the following statements, specify "True" if the statement is always true. Otherwise specify "False."
   a. To prove "Uniqueness" is to prove "There is at most one … ."
   b. Perpendicularity is transitive.
   c. If a diagonal of a parallelogram bisects two of the angles of the parallelogram, then the parallelogram is a rectangle.
   d. The way to start an indirect proof is to assume that part of the given information is false.

3. The angles of a quadrilateral are in the ratio $4 : 5\frac{1}{2} : 6\frac{1}{2} : 8$. The measure of the smallest angle of the quadrilateral is
   a. 45   b. 50   c. 55   d. 60

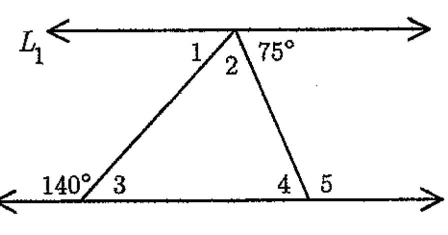
4. In ΔPQR, $\angle P \cong \angle R$. $PQ = 2x+1$, $QR = 3x-2$, and $PR = 5x-4$. The perimeter of ΔPQR is
   a. 24   b. 25   c. 26   d. 27

5. Given: $\overline{AH}$ bisects $\angle BAC$.
   $m\angle B = 40$, $m\angle D = 20$, $m\angle ACD = 100$.

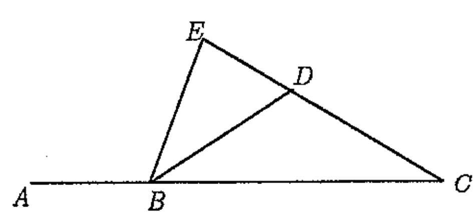
   Find: a. $m\angle MAE$
   b. $m\angle AME$



6. In the diagram with $L_1 \parallel L_2$ and the angles as marked, find the measures of the numbered angles.



7. In the figure to the right, $BE = BD = DC$ and $m\angle C = 35$. Find $m\angle EBA$.



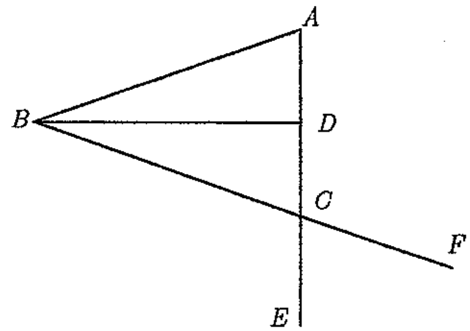
8. Prove that if ΔABC is isosceles with altitudes $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$, then ΔXYZ is also isosceles.

9. In ΔABC, the bisectors of $\angle A$ and $\angle C$ meet at P. If $m\angle B = 100$, find $m\angle APC$.

10. In isosceles ΔABC, M is the midpoint of the base $\overline{AB}$. P is on $\overline{AC}$ and Q is on $\overline{BC}$ so that $\angle AMP \cong \angle BMQ$. Prove that $\angle MPC \cong \angle MQC$.

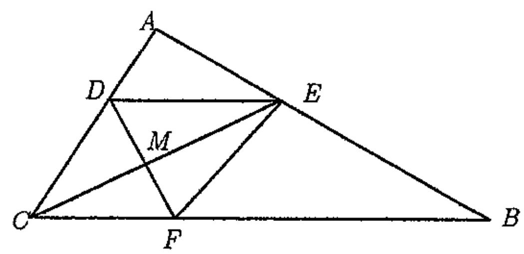
11. Given: $\overline{AD} \cong \overline{CD}$
$\angle ECF \cong \angle BAD$

Prove: $\angle ABD \cong \angle CBD$



12. Given: ΔABC with $\overline{CE}$ bisecting $\angle ACB$.
$\overline{DF}$ is the perpendicular bisector of $\overline{CE}$.

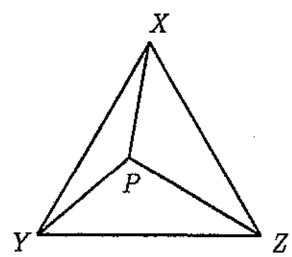
Prove: $CDEF$ is a rhombus.



13. Prove indirectly: If, in ΔABC, E is the midpoint of $\overline{AB}$ and F is the midpoint of $\overline{AC}$, then $\overline{BF}$ and $\overline{EC}$ do not bisect each other.

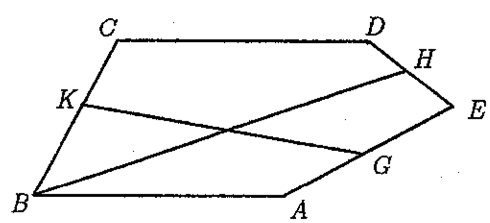
14. Given: ΔXYZ is equilateral;
$\angle PXY \cong \angle PYX$.

Prove: $\overline{ZP}$ bisects $\angle XZY$.



15. Given: $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$.
G, H, and K are the midpoints of $\overline{AE}$, $\overline{ED}$, and $\overline{BC}$, respectively.

Prove: $\overline{GK}$ and $\overline{BH}$ bisect each other.



16. Given: In ΔABC, points D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$, respectively.

Prove: $m\angle BDE > m\angle C$

Cumulative Review
Number 3

1. For each of the following hypotheses, draw a conclusion and state (or identify) a theorem, corollary, or postulate you would use to justify your conclusion.
   a. In ΔABC, if M is the midpoint of $\overline{AB}$ and N is the midpoint of $\overline{BC}$, then ... .
   b. If ΔABC is isosceles with base $\overline{BC}$ and $\overline{AP}$ is an altitude of the triangle, then ... .
   c. If $\overline{PM} \perp \overline{AB}$ and M is the midpoint of $\overline{AB}$, then ... .
   d. In a plane, if $\overline{AB} \perp \overline{BD}$ and $\overline{BD} \perp \overline{CD}$, then ... .

2. For each of the following statements, specify "True" if the statement is always true. Otherwise specify "False."
   a. The Parallel Postulate is a statement about uniqueness.
   b. If the diagonals of a quadrilateral are congruent, then the quadrilateral is a rectangle.
   c. If C is on $\overline{AB}$ and P is not on $\overline{AB}$, and $AP = BP$, then $\overleftrightarrow{PC}$ is the perpendicular bisector of $\overline{AB}$.
   d. If $\overline{PX} \perp \overline{RS}$ at X and $PR = PS$, then $RX = SX$.

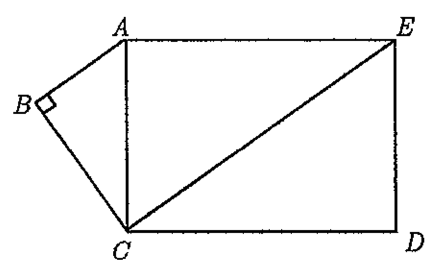
3. The sum of the measures of the interior angles of a convex polygon is double the sum of the measures of the exterior angles. How many sides does the polygon have?
   a. 4 b. 5. c. 6. d. 8

4. In quadrilateral $WXYZ$, the sides have lengths $WX = 7$, $XY = 10$, $YZ = 12$, and $ZW = 11$. Which of the following might be the length of diagonal $\overline{WY}$?
   a. 2 b. 15 c. 18 d. all of these

5. $ABWXYZ$ is a regular hexagon and $ABCDE$ is a regular pentagon inside it. Then $m\angle CBW =$
   a. 9 b. 12 c. 15 d. 18

6. In a regular polygon, the ratio of the measure of an interior angle to the measure of an exterior angle is $13:2$. How many sides does the polygon have?
   a. 10 b. 12 c. 15 d. 18

7. In the diagram, $ABCE$ is a trapezoid with $\overline{AB} \parallel \overline{CE}$, $ACDE$ is a rectangle, and $\angle B$ is a right angle.
   a. Name two complements of $\angle ACE$.
   b. Name two angles congruent to $\angle ACE$ other than $\angle ACE$ itself.
   c. Name a supplement of $\angle B$.



8. Eight times the measure of an angle is equal to the sum of the measures of the angle's complement and its supplement. Find the measure of the angle.

9. In ΔABC, $AC = BC$. If $m\angle C = \frac{1}{2} m\angle B$, what is $m\angle A$?

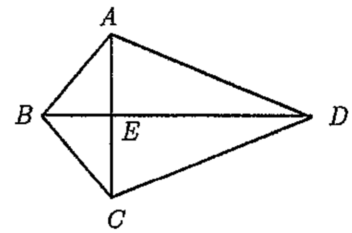
10. In ΔABC, $m\angle ABC = 90$ and $m\angle C = 36$. $\overline{BM}$ is a median and $\overline{BE}$ is the altitude to $\overline{AC}$. Find $m\angle EBM$.

11. Find the length of the segment joining the midpoints of the two longest sides of a triangle whose sides have lengths 12, 13, and 14.

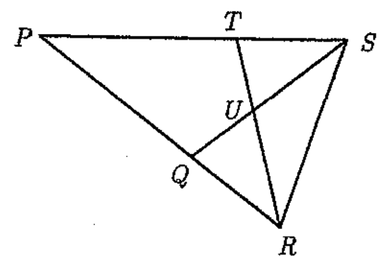
12. In ΔABC, $AB = 6$ and $BC = 9$. The length of $\overline{AC}$ must satisfy $x < AC < y$. Determine x and y.

13. Given: In ΔABC, $\overline{AB} \cong \overline{BC}$. E is the midpoint of $\overline{AC}$.

Prove: ΔADC is isosceles.



14. In the figure, $\overline{RU} \cong \overline{SU}$ and $\overline{QU} \cong \overline{TU}$. Prove that ΔPRS is isosceles.

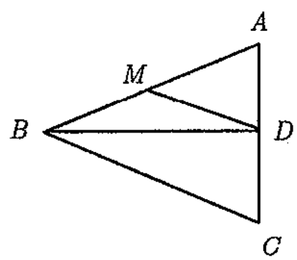


15. $\overline{BD}$ is an altitude of ΔABC with D between A and C, but not the midpoint of $\overline{AC}$. Prove that $AB \neq BC$.

16. In parallelogram $ABCD$, the bisectors of $\angle A$ and $\angle C$ intersect diagonal $\overline{BD}$ at X and Y, respectively. Prove that $AXCY$ is a parallelogram.

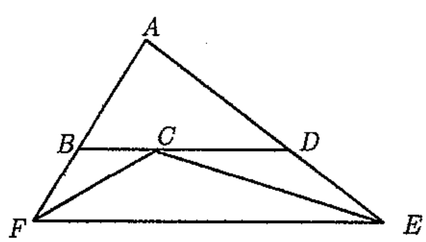
17. Given: $\overrightarrow{BD}$ bisects $\angle ABC$
$\overline{BD} \perp \overline{AC}$
$\overline{MD} \parallel \overline{BC}$

Prove: $AM = MB$

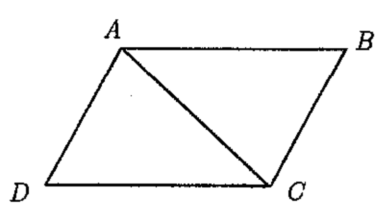


18. Given: $\overline{FC}$ bisects $\angle AFE$;
$\overline{EC}$ bisects $\angle AEF$;
$\overline{BCD} \parallel \overline{FE}$.

Prove: $BD = BF + DE$.



19. Given $\overline{AB} \cong \overline{DC}$;
$\overline{AD} \cong \overline{BC}$;
$AD < AB$.



Prove: $\overline{AC}$ does not bisect $\angle BAD$

Cumulative Review
Number 4

1. For each of the following hypotheses, draw a conclusion and state (or identify) a theorem, corollary, or postulate you would use to justify your conclusion.
   a. If $\angle ABC$ and $\angle ABD$ are adjacent and A is between C and D, then … .
   b. If C and D are points on opposite sides of $\overleftrightarrow{AB}$ and $\angle CBA \cong \angle DAB$, then … .
   c. If $\angle A$ is a supplement of $\angle B$ and $\angle B$ is a supplement of $\angle C$, then … .
   d. If $\angle A$ is congruent to its supplement, then … .

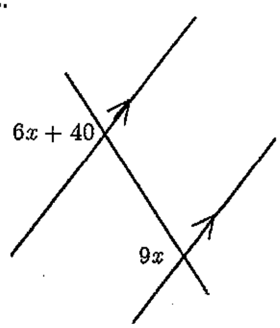
2. For each of the following statements, specify "True" if the statement is always true. Otherwise specify "False."
   a. If $AB = BC$, then B is the midpoint of $\overline{AC}$.
   b. Proving "There is exactly one" means proving both existence and uniqueness.
   c. In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other one as well.
   d. If the diagonals of a quadrilateral are perpendicular, then the quadrilateral is a rhombus.

3. In ΔABC, the measure of $\angle B$ is twice the sum of the measures of $\angle A$ and $\angle C$. Find the measure of the exterior angle at B.

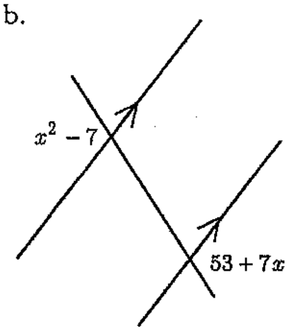
4. In ΔABC, AB is 5 more than twice BC and AC is 3 less than twice BC. Show that $BC > 8$.

5. Determine the value of each variable.

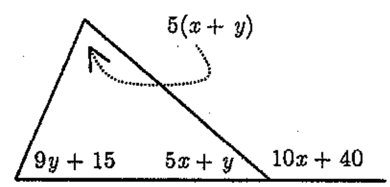
a.



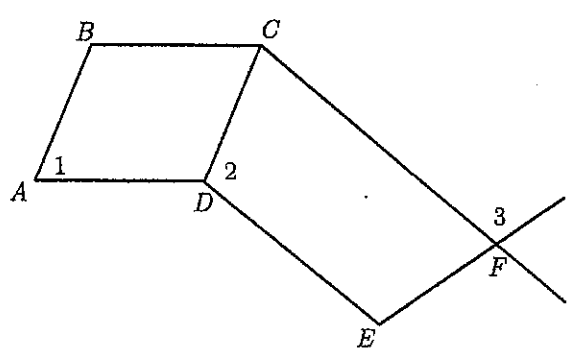
b.



c.



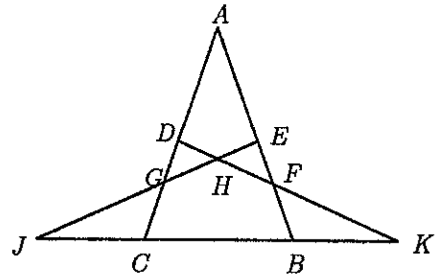
6. In the figure, $ABCD$ is a parallelogram and $CDEF$ is an isosceles trapezoid. If $m\angle ABC = 110$ and $m\angle ADE = 130$, determine the measures of $\angle 1$, $\angle 2$, and $\angle 3$.



7. In quadrilateral $ABCD$, $m\angle A = x$, $m\angle B = 4x$, $m\angle C = 5x$, and $m\angle D = 8x$. Determine the measures of the four angles and which sides must be parallel.

8. Given: $\overline{AC} \cong \overline{AB}$; $\overline{JC} \cong \overline{KB}$;
$\angle JGC \cong \angle KFB$.

Prove: $\overline{JE} \cong \overline{KD}$.



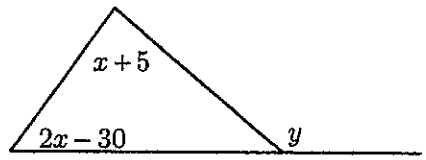
9. Given the figure with angle measures as marked. Determine whether the following statements are true or false.

a. $x > 15$

b. $y > 3x - 25$

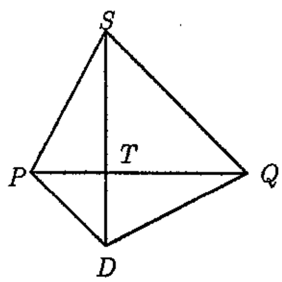
c. $x < 15 + \frac{y}{2}$

d. $3x - 2y < 25$



10. Given: $\overline{PQ} \perp \overline{DS}$, $PS = DQ$,
$\angle DSQ \cong \angle PQS$

Prove: $PT = DT$

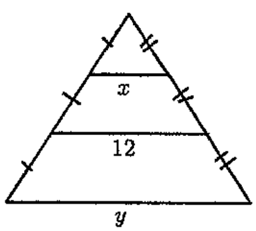


11. The measure of one angle of a triangle is 3 less than twice the measure of a second angle. The measure of the third angle is double the sum of the other two angles. What are the measures of the angles of the triangle?

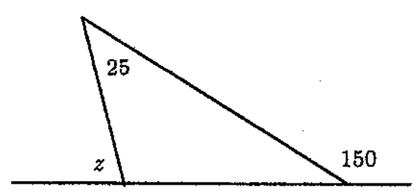
12. The measure of an interior angle of a regular polygon is 14 times the measure of an exterior angle of the same polygon. How many sides does the polygon have?

13. Find the values of each variable.

a.



b.



14. $WXYZ$ is a rectangle. Points A, B, C, and D are the midpoints of $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, and $\overline{ZW}$, respectively. Prove that $ABCD$ is a rhombus.

15. $ABCDE$ is a regular pentagon and $DEFG$ is a square. Find $m\angle EAF$ and $m\angle AFD$.