Exercises 3.1

1. If you were writing an **indirect proof** of the following statements, what assumption would you begin with?

   a. If x^2 is even, then x is even.

   b. A triangle has at most one right angle.

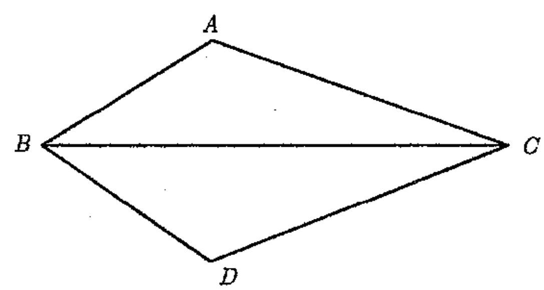
   c. If $ab = 0$, then $a = 0$ or $b = 0$.

2. Prove that the bisector of any angle of a scalene triangle cannot be perpendicular to the opposite side.

3. Prove indirectly:

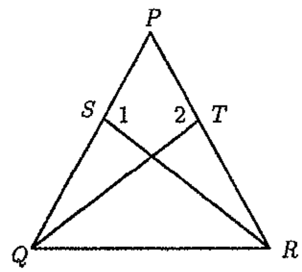
   <u>Given</u>: $BA = BD$
   $CA \neq CD$

   <u>Prove</u>: $\overline{BC}$ does not bisect $\angle ABD$



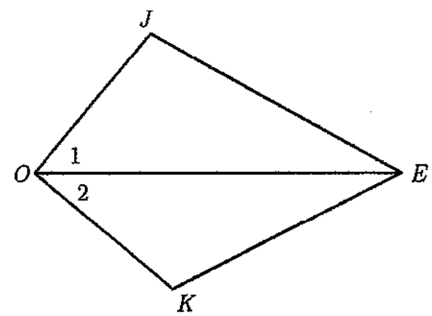
4. Given: $PR = PQ$
$QT \neq RS$

Prove: $\angle 1 \not\cong \angle 2$



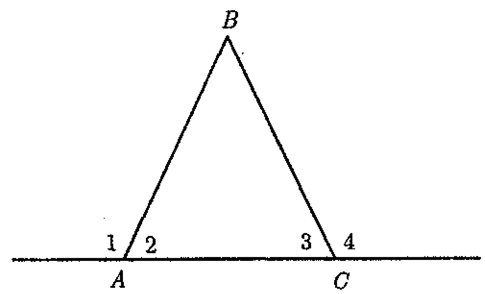
5. Given: $\angle 1 \not\cong \angle 2$
$\overline{OJ} \cong \overline{OK}$

Prove: $\angle J$ and $\angle K$ are not both right angles.



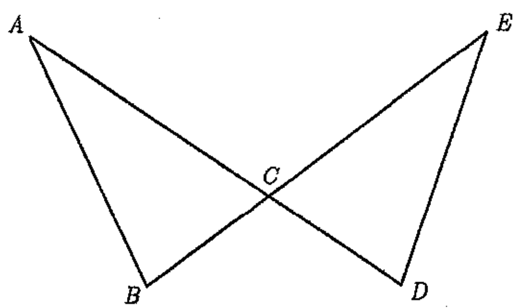
6. Given: $\angle 1 \not\cong \angle 4$

Prove: B does not lie on the perpendicular bisector of $\overline{AC}$.



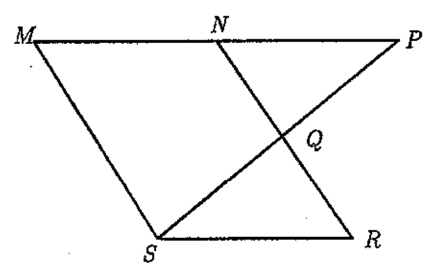
7. Given: $AB = ED$
$AC \neq EC$

Prove: $\angle B \not\cong \angle D$



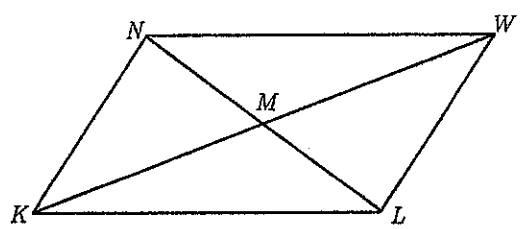
8. Given: N is the midpoint of $\overline{MP}$
$\angle P \cong \angle PSR$
$MN \neq SR$

Prove: Q is not the midpoint of $\overline{PS}$



9. Given: M is the midpoint of $\overline{NL}$
$KN \neq KL$

Prove: $NW \neq LW$

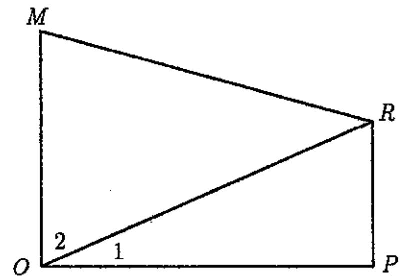


Exercises 3.2

1. Prove **Theorem 3.3** directly: In a plane, if two lines are perpendicular to a third line, then they are parallel to each other.

2. Given: $\angle 1$ is complementary to $\angle 2$ and $\overline{RP} \perp \overline{OP}$

   Prove: $\overline{MO} \parallel \overline{RP}$

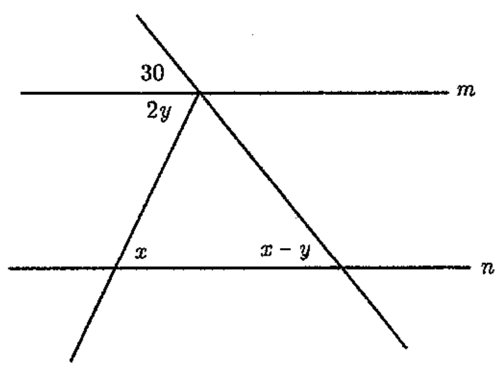


3. Prove **Theorem 3.3** indirectly: In a plane, if two lines are perpendicular to a third line, then they are parallel to each other.

4. Prove **Theorem 3.5**: Given two coplanar lines and a transversal, if two corresponding angles are congruent, then the lines are parallel.

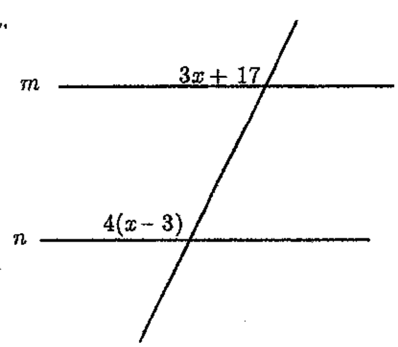
5. Prove **Theorem 3.6** directly: Given two coplanar lines and a transversal, if twp same-side interior angles are supplementary, then the lines are parallel.

6. Find the values of x and y that make $m \parallel n$.

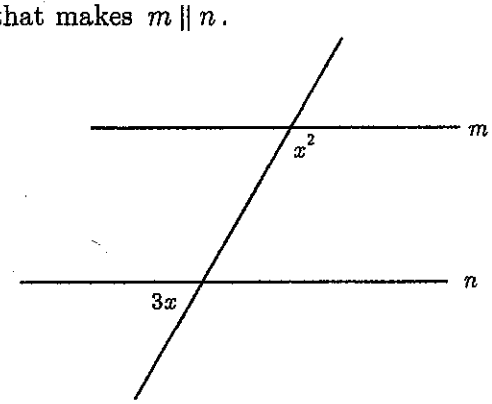


7. In each of the following find the value of x that makes $m \parallel n$.

a.

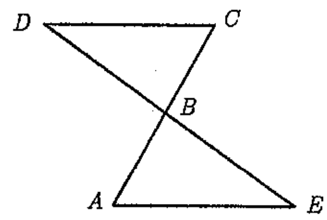


b.

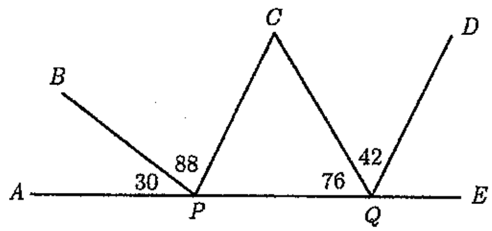


8. Given: $\overline{AC}$ and $\overline{DE}$ bisect each other.

Prove: $\overline{AE} \parallel \overline{CD}$

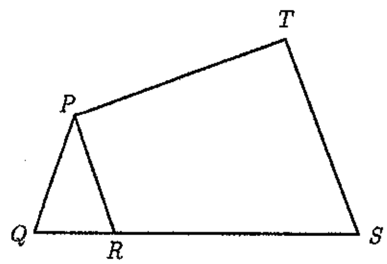


9. In the diagram, why is $\overline{PC}$ parallel to $\overline{QD}$?



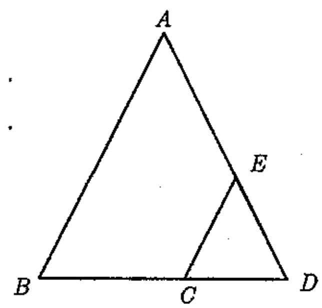
10. Given: $\overline{PQ} \cong \overline{PR}$
$\angle Q \cong \angle S$

Prove: $\overline{PR} \parallel \overline{ST}$



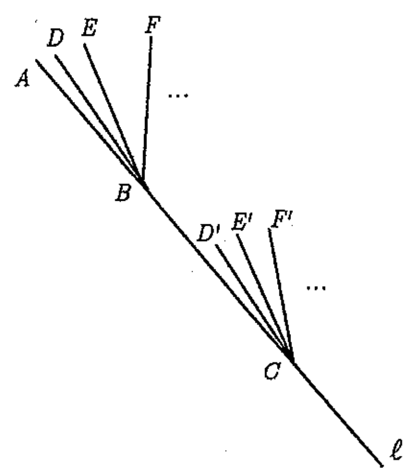
11. Given: A is on the perpendicular bisector of $\overline{BD}$.
E is on the perpendicular bisector of $\overline{CD}$.

Prove: $\overline{AB} \parallel \overline{CE}$



12. In ΔPQR the median $\overline{PM}$ is extended through M to S so that $PM = MS$. Prove that $\overline{PQ} \parallel \overline{RS}$.

13. In the figure, $\overrightarrow{BD}$, $\overrightarrow{BE}$, $\overrightarrow{BF}$, and so on, form angles with line ℓ, with $m\angle ABD = 1$, $m\angle DBE = 2$, $m\angle EBF = 4$, $m\angle FBG = 8$, ..., doubling each time. Also, $\overrightarrow{CD'}$, $\overrightarrow{CE'}$, $\overrightarrow{CF'}$, and so on, form angles with line ℓ with $m\angle BCD' = 1$, $m\angle D'CE' = 2$, $m\angle E'CF' = 3$, $m\angle F'CG' = 4$, ..., increasing by one each time. All rays are on the same side of $\overleftrightarrow{ABC}$. Determine all the pairs of parallel rays.



Exercises 3.3

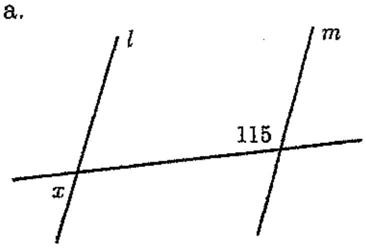
1. Prove **Theorem 3.8**: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

2 Prove **Theorem 3.9**: If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.

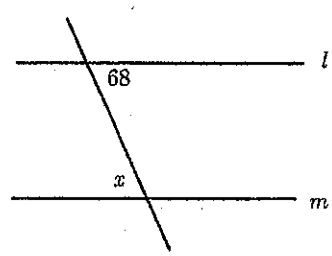
3. Prove that if a line is perpendicular to one of two parallel lines then it is perpendicular to the other.

4. Given that $l \parallel m$, find the value of x in each of the following.

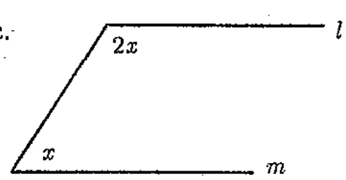
a.



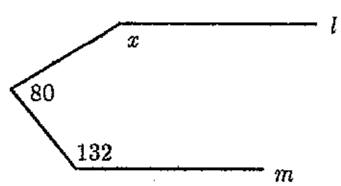
b.



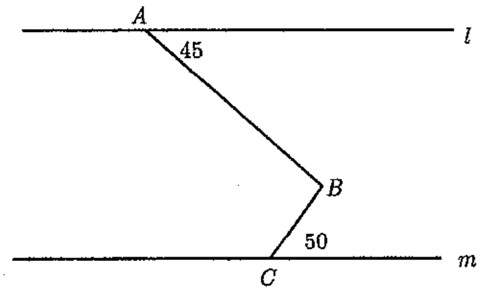
c.



d.

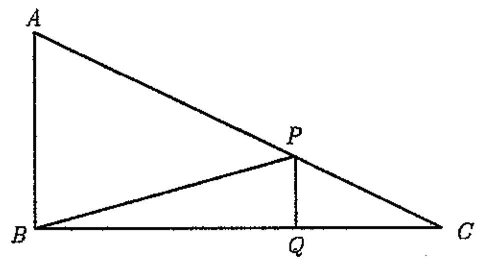


5. l is parallel to m. Find $m\angle ABC$.



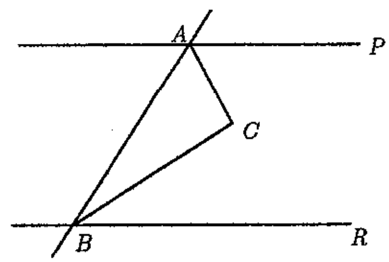
6. Given: $\overline{PQ} \perp \overline{BC}$
$\overline{AB} \perp \overline{BC}$

Prove: $\angle ABP \cong \angle BPQ$



7. Given: $\overline{AP} \parallel \overline{BR}$
$\overline{AC}$ bisects $\angle BAP$.
$\overline{BC}$ bisects $\angle ABR$.

Prove: $\angle CAB$ and $\angle CBA$ are complementary angles.

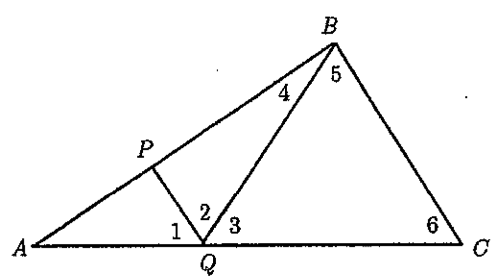


8. Prove that if a line is parallel to the base of an isosceles triangle and intersects the other two sides in two different points, then an isosceles triangle is formed.

9. In ΔABC, the bisector of $\angle A$ intersects $\overline{BC}$ at D. The perpendicular bisector of $\overline{AD}$ intersects $\overline{AC}$ at G. Prove that $\overline{GD} \parallel \overline{AB}$.

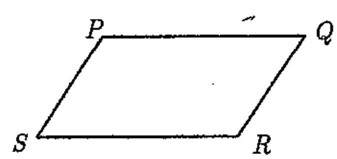
10. Given: $\overline{PQ} \parallel \overline{BC}$
$\angle 5 \cong \angle 6$

Prove: $\overline{QP}$ bisects $\angle AQB$



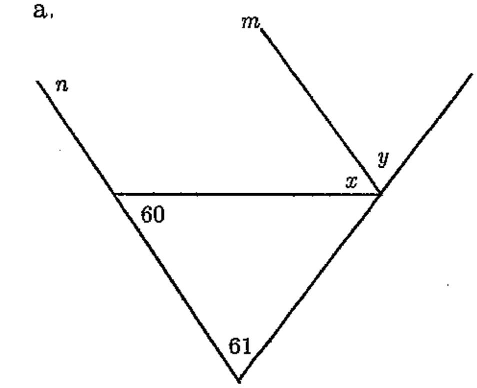
11. Given: In quadrilateral $PQRS$, $\overline{PQ} \cong \overline{RS}$ and $\overline{PQ} \parallel \overline{RS}$.

Prove: $\overline{PS} \parallel \overline{QR}$

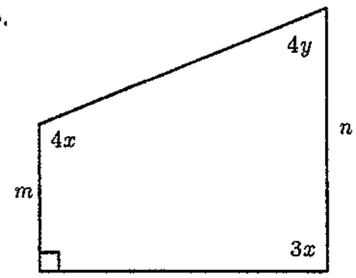


12. In each of the following, find the values of x and y that make the lines m and n parallel.

a.

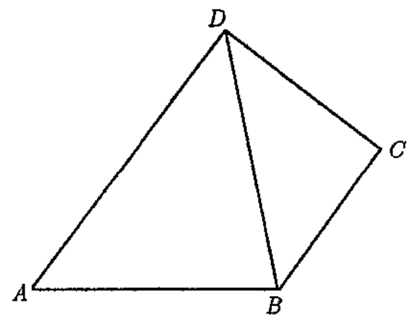


b.



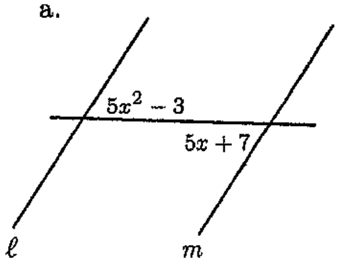
13. <u>Given:</u> $\overline{DB}$ bisects $\angle ADC$.
$\overline{AD} \parallel \overline{BC}$

<u>Prove:</u> $\overline{BC} \cong \overline{CD}$

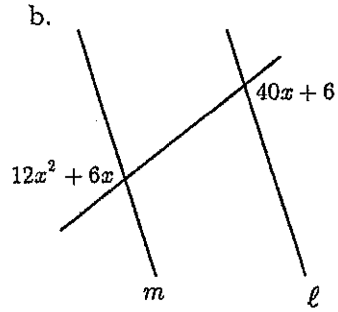


14. In each of the figures below, $\ell \parallel m$. Determine all values that x can take on.

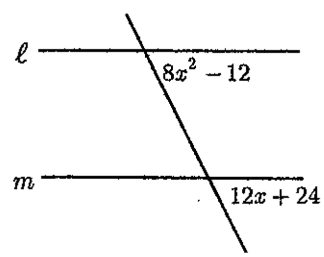
a.



b.

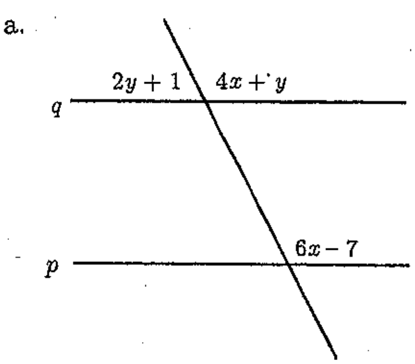


c.

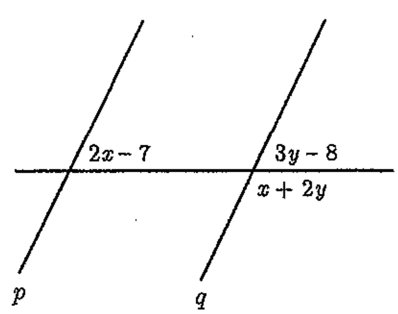


15. In each of the figures below, $p \parallel q$. Determine the values of x and y.

a.



b.



Exercises 3.4

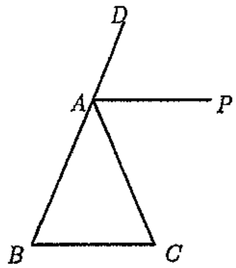
1. Prove **Theorem 3.11**: The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

2. Prove **Theorem 3.12**: If two angles of one triangle are congruent, respectively, to two angles of a second triangle, then the third angles are congruent.

3. The measures of the angles of a triangle are in the ratio $1:2:3$. Find the measure of each angle of the triangle.

4. The measure of one angle of a triangle is 25 more than the measure of a second angle of the triangle and the measure of the third angle is 9 less than twice the measure of the second angle. Find the measure of each angle.

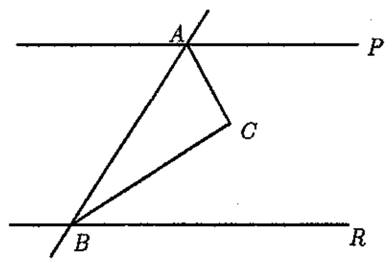
5. Given that ΔABC is isosceles with $AB = AC$. $\overline{AP}$ bisects $\angle DAC$. Prove that $\overline{AP}$ is parallel to $\overline{BC}$.



6. $\overline{AM}$ is a median of ΔABC. Through C a line is drawn parallel to $\overline{AM}$ and intersecting $\overleftrightarrow{AB}$ at P. Prove that $\overline{AB} \cong \overline{AP}$.

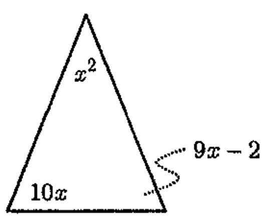
7. Given: $\overline{AP} \parallel \overline{BR}$
   $\overline{AC}$ bisects $\angle BAP$.
   $\overline{BC}$ bisects $\angle ABR$.

   Prove: $\angle C$ is a right angle.

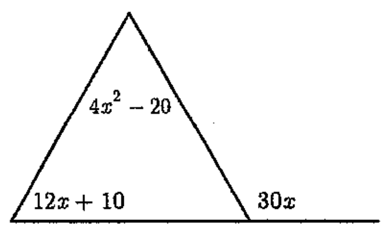


8. Determine the measures of the angles in each of the figures below.

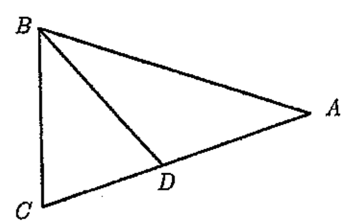
a.



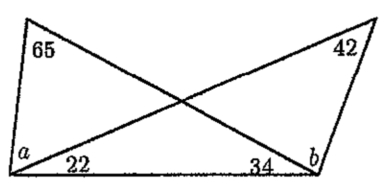
b.



9. Pictured to the right is isosceles ΔABC with $m\angle A = 36$. $\overline{BD}$ bisects $\angle ABC$. If $AB = AC = 1$ and $BC = x$, then determine:
   a. the measure of each angle in the figure;
   b. the length of $\overline{CD}$ in terms of x.

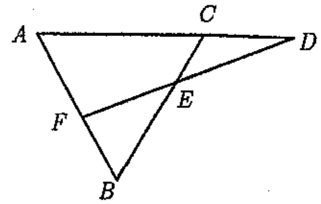


10. In the figure, determine a and b.



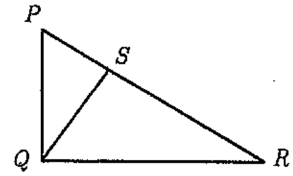
11. Given: $\angle B \cong \angle ACB$
$\angle D \cong \angle CED$

Prove: $m\angle AFD = 3 \cdot m\angle D$



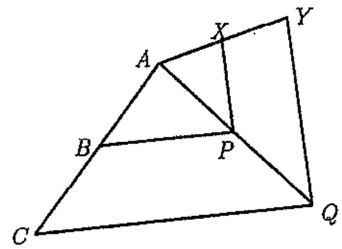
12. Given: In ΔPQR, $\angle PQR$ is a right angle.
$\overline{QS} \perp \overline{PR}$.

Prove: $\angle P \cong \angle SQR$

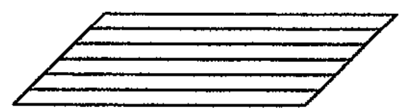


13. Given: B is the midpoint of $\overline{AC}$;
$\overline{BP} \parallel \overline{CQ}$; $\overline{PX} \parallel \overline{QY}$.

Prove: $AX = XY$

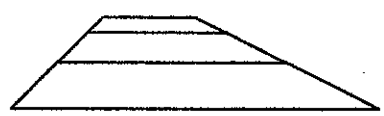


14. In the figure, the seven horizontal lines are parallel and each of the six individual segments on the left has length 1.



a. Why is the total length of the segment on the right not necessarily 6?
b. Can the lengths of the six individual pieces on the right all be different? Explain.

15. In the figure, the four horizontal lines are parallel, and the three individual segments on the left-hand transversal have lengths 1, 2, and 3.



If the total length of the right-hand transversal is 12, what are the lengths of the three individual pieces? How could the addition of some extra parallel lines make this clear?

Exercises 3.5

1. a. Prove that an equilateral triangle is also equiangular.
   b. Is an equilateral triangle a regular polygon?
   c. What is the measure of each interior angle of an equilateral triangle?

2. If a convex polygon has n sides, find the number of triangles formed if all of the diagonals from a single vertex are drawn.
   a. $n = 4$ b. $n = 5$ c. $n = 6$ d. $n = 12$ e. $n = x$

3. Find the sum of the measures of all of the interior angles of a convex pentagon (5 sides) and a convex hexagon (6 sides).

4. What is the sum of the measures of all the exterior angles of convex 15-gon? If the 15-gon is regular, what is the measure of each exterior angle?

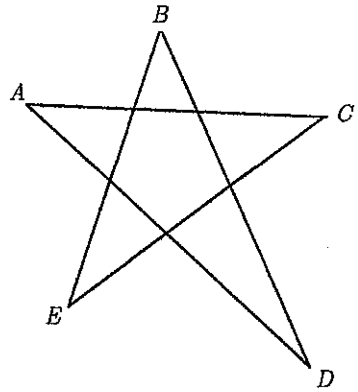
5. a. If a regular polygon has 12 sides, what is the sum of the measures of all of the interior angles?
   b. If a regular polygon has 12 sides, what would be the measure of each interior angle of the polygon?

6. The sum of the measures of the interior angles of a convex polygon is four times the sum of the measures of its exterior angles. How many sides does the polygon have?

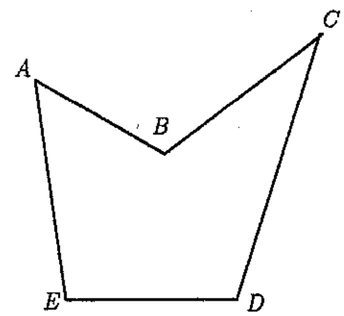
7. $ABCDEFGHIJ$ is a regular decagon (10-sided polygon). If $\overline{AB}$ and $\overline{CD}$ are extended to meet at P, find $m\angle P$.

8. In quadrilateral $ABCD$, $m\angle A = x$, $m\angle B = 2x$, $m\angle C = 3x$, and $m\angle D = 4x$. Find the value of x and determine if any sides of quadrilateral $ABCD$ must be parallel.

9. Find $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E$.



10. We have stated that the theorems that we have developed will apply to convex polygons.
   a. Consider the nonconvex polygon $ABCDE$ whose interior angle at B is greater than 180. Can you show that the sum of all of the interior angles of $ABCDE$ **still** is 540?



   b. Is it true that the sum of the measures of the angles of a nonconvex polygon of n sides is $(n-2)\cdot 180$?

   c. Is the sum of the exterior angles of a nonconvex polygon equal to 360? Explain.

11. A pentagon has its angles in the ratio $2:3:2:3:2$ in order going around. What are the measures of the five angles? Draw a reasonably accurate figure of this pentagon.

12. The exterior angles of a hexagon are in the ratio $1:2:3:2:1:3$ in order, going around. What are the angles of the hexagon? Draw a reasonably accurate picture of this hexagon.

13. The angles of an octagon are in the ratio $9:8:7:6:6:7:8:9$ in order, going around. What are the angles of the octagon?

14. A geometry book said that the six angles of a convex hexagon were in the ratio $1:2:3:4:5:6$. When a student tried the problem, she decided there had to be a typographical error in it. Why?

15. Prove Nanda's Theorem: If each interior angle of a regular polygon has a measure that is k times the measure of an exterior angle, then the polygon has $2(k+1)$ sides.

16. The average of the measures of an interior angle and an exterior angle of a regular polygon is 9 times the number of sides in the polygon. How many sides does the polygon have?

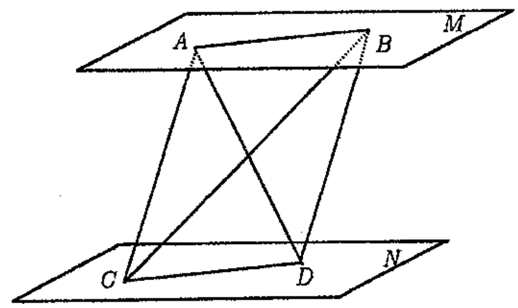
17. Two regular polygons have interior angles with measures that are integers differing from each other by 10. How many sides do the polygons have? (There are six solutions!)

Exercises 3.6

1. Prove that two lines that are each perpendicular to the same plane are parallel to each other.

2. Given: Planes M and N are parallel;
$\overline{AB}$ lies in M and $\overline{CD}$ lies in N;
A, B, C, and D are coplanar;
$\overline{AB} \cong \overline{CD}$.

Prove: $\overline{AD}$ and $\overline{BC}$ bisect each other.

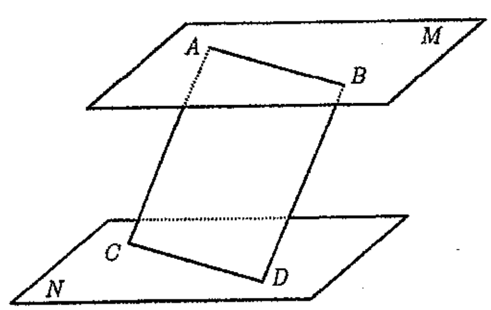


3. True or false? If M and N are parallel planes and M contains $\overleftrightarrow{AB}$ while N contains $\overleftrightarrow{CD}$, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

4. For each of the following, decide whether the statement is true or false. Draw a diagram to illustrate your conclusion in each case.
   a. If a line lies in a plane, then any line parallel to that line is parallel to the plane.
   b. If a line and a plane are parallel, then every line in the plane is parallel to the given line.
   c. Two lines parallel to the same plane may be perpendicular to each other.
   d. If two lines are parallel, then every plane containing only one of the lines is parallel to the other line.
   e. If a plane intersects two parallel planes, the lines of intersection are parallel.
   f. If a plane intersects two intersecting planes, the lines of intersection may be parallel.

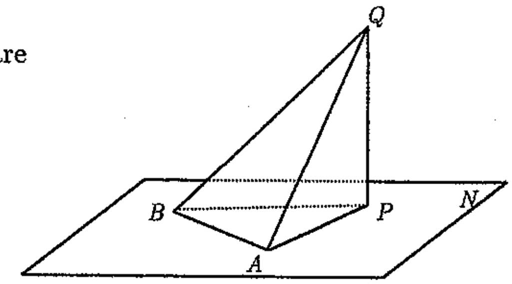
5. Given: $\overline{AB} \cong \overline{CD}$; plane $M \parallel$ plane N;
$\overrightarrow{AC} \parallel \overrightarrow{BD}$.

Prove: $\overline{AC} \cong \overline{BD}$



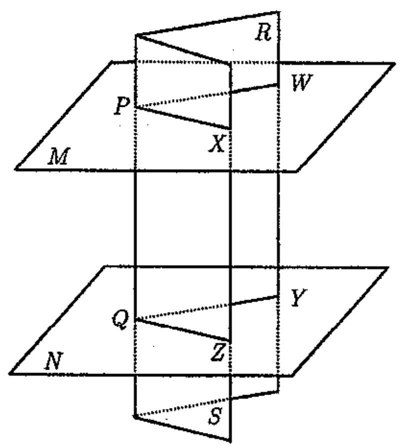
6. Given: $\overline{QP} \perp$ plane N at P. A and B are in plane N with $AP = BP$.

Prove: $\angle ABQ \cong \angle BAQ$



7. Prove (indirectly): If a line is perpendicular to both plane M and plane N, then plane $M \parallel$ plane N.

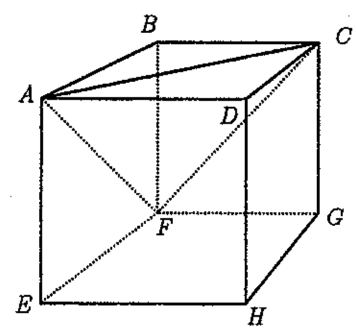
8. Let plane M and plane N both be perpendicular to $\overline{PQ}$. Let plane R and plane S be (noncoplanar) planes that contain $\overline{PQ}$. Let W and X both be in plane M, and lying in planes R and S, respectively. Let Y and Z both be in plane N, and lying in planes R and S, respectively. Prove that if $\overline{PW} \cong \overline{QY}$ and $\overline{PX} \cong \overline{QZ}$, then $\overline{WX} \cong \overline{YZ}$.



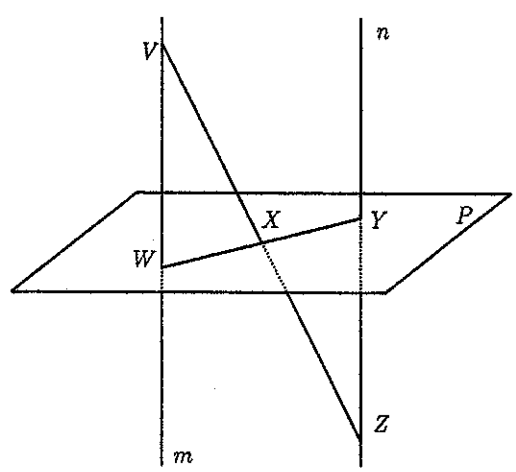
9. Given the cube pictured. (A cube is a solid with the following properties: the twelve edges are congruent to each other; any two edges that intersect are perpendicular to each other.) Find each of the following angle measures.

a. $m\angle FAE$ b. $m\angle FCB$

c. $m\angle BAF$ d. $m\angle ACF$



10. In the figure, line $m \perp$ plane P at W and line $n \perp$ plane P at Y. Also, $\overline{VW} \cong \overline{YZ}$. Explain why X is the midpoint of $\overline{VZ}$.



Exercises 3.7

1. Determine which of the following can be the lengths of the three sides of a triangle.
   a. $a = 3$, $b = 4$, $c = 7$ b. $a = 5$, $b = 10$, $c = 5$
   c. $a = 10$, $b = 3$, $c = 6$ d. $a = 11$, $b = 12$, $c = 2$

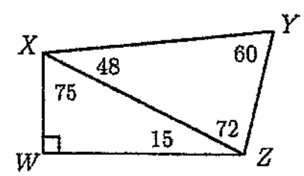
2. The lengths of two of the sides of a triangle are 5 and 8. If the third side has length x, then between what two numbers must x lie?

3. In ΔPQR, $PQ = 5$, $PR = 7$, and $QR = 8$. Arrange the angles in order from smallest to largest.

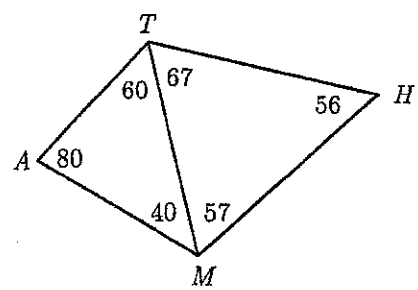
4. In ΔABC, $AB = 11$, $BC = 17$, and $AC = x$. What is the range of values that x can take on?

5. In ΔABC, $AB = 11$, $BC = 17$, and $AC = 2x - 3$. How many integer values can x have?

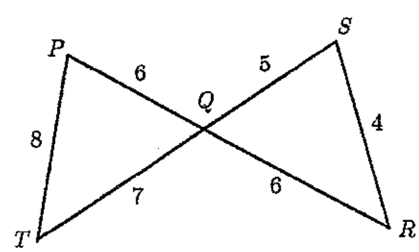
6. In the figure to the right, which is the longest segment? Why can't you determine which segment is shortest?



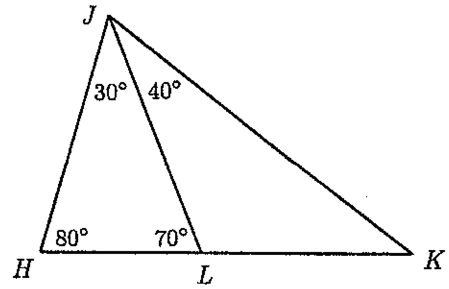
7. Arrange the lengths of the five segments in the figure to the right in order from shortest to longest.



8. One might think that the idea behind problems 6 and 7 (using Theorem 3.19) could be used just as well with sides given (calling into play Theorem 3.18). What is wrong with the diagram to the right?



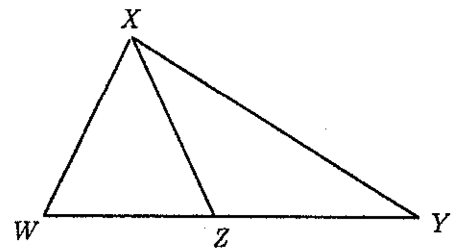
9. Given ΔHJK with angle measures as marked. Determine, if possible, the longest and shortest segments in the figure.



10. The bisector of $\angle CAB$ in ΔABC meets $\overline{BC}$ at D. Prove that $AC > DC$.

11. Given: $WX = XZ$

Prove: $XY > XW$



12. Use the Triangle Inequality to prove that if a point P is taken in the interior of a triangle, the sum of the lengths of the segments drawn from P to the vertices of the triangle is greater than half the perimeter of the triangle.

13. Prove: If D is any point in the interior of ΔABC, then $AB + BC > AD + DC$. [Hint: Extend $\overline{AD}$ to E on $\overline{BC}$ and prove $AB + BC > AE + EC$.] Write a paragraph proof.

14. Prove: If the base of an isosceles triangle is the longest side of the triangle, then the vertex angle has a measure greater than 60. Write a paragraph proof.

15. In ΔPQR, $\overline{RS}$ is the altitude to $\overline{PQ}$. Prove that $\frac{PR + RQ}{2} > RS$.

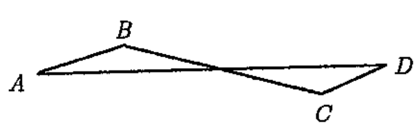
16. Prove that a segment drawn from the vertex of an isosceles triangle to any point in the base is shorter than either of the congruent sides.

17. The lengths of the sides of a triangle are a, b, and x, with $a > b$. Determine the limiting values of x in terms of a and b.

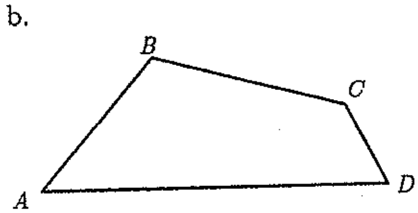
18. Prove that the difference of the lengths of any two sides of a triangle is less than the measure of the third side.

19. For each of the situations pictured below, explain why $AD < AB + BC + CD$.

a.

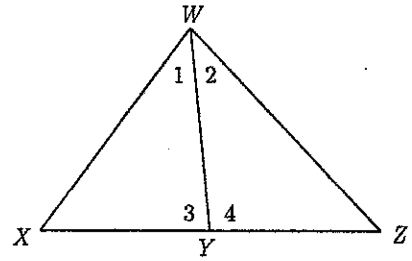


b.

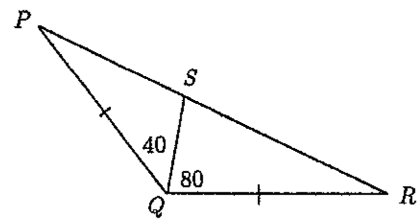


Exercises 3.8

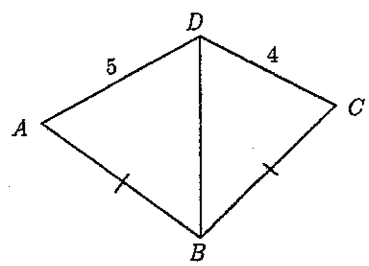
1. What can you conclude from the given information in each of the following?
   a. $\angle 1 \cong \angle 2$ and $WZ > WX$.
   b. Y is the midpoint of $\overline{XZ}$ and $m\angle 4 > m\angle 3$.
   c. $\overline{WY}$ is a median of ΔWXZ and $WX > WZ$.
   d. W is on the perpendicular bisector of $\overline{XZ}$ and $m\angle 1 > m\angle 2$.



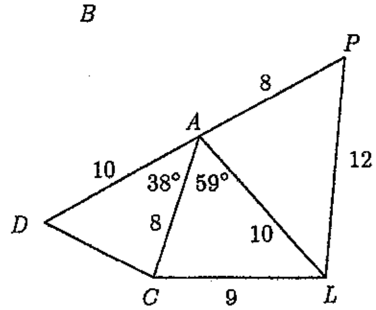
2. Given the figure as marked, prove that $SR > PS$.



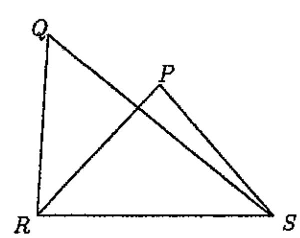
3. In the figure to the right, how do you know that $m\angle ABD > m\angle CBD$?



4. In the figure to the right, lengths of six segments and measures of two angles are given. (Do **not** assume D, A, and P are collinear.)
   a. What can you say about DC? Why?
   b. What can you say about $m\angle PAL$? Why?

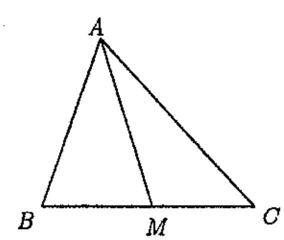


5. In the figure, point P is in the interior of $\angle QRS$ and $RQ = RP$. Prove that $QS > PS$.



6. Given: In ΔABC, $\overline{AM}$ is the median to $\overline{BC}$. $m\angle AMC > m\angle AMB$.

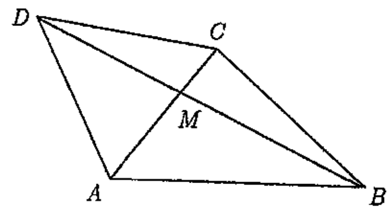
   Prove: $m\angle B > m\angle C$



7. In ΔXYZ, $XZ > YZ$ and P is the midpoint of $\overline{XY}$. Is $\angle ZPX$ acute or obtuse? Explain your reasoning.

8. Given: In the figure, $AD = BC$, $DM > AM$, and $BM > CM$.

Prove: $m\angle DAB > m\angle CBA$

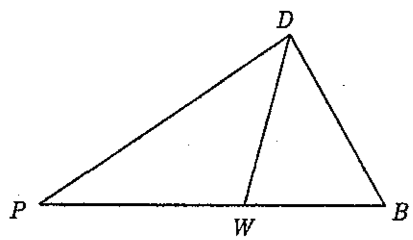


9. Median $\overline{BD}$ is drawn to the hypotenuse of right triangle ΔABC. Prove that if $m\angle BDC > m\angle BDA$, then $m\angle A > m\angle C$.

10. In ΔABC, $\overline{BD}$ and $\overline{BE}$ are respectively the altitude and the median to side $\overline{AC}$. Prove that if point E lies between points D and C, then $m\angle A > m\angle C$.

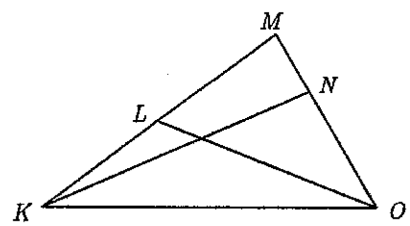
11. Given: $PW = DB$

Prove: $PD > WB$



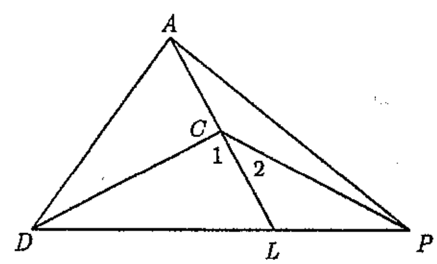
12. Given: $KM > OM$
$KL = ON$

Prove: $KN > OL$



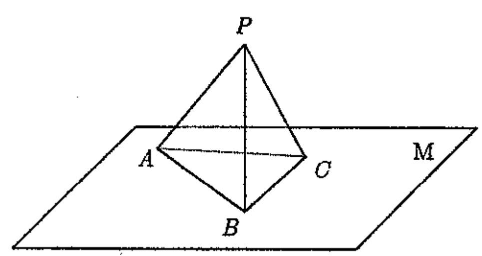
13. Given: $\angle CDP \cong \angle CPD$
$m\angle 1 > m\angle 2$

Prove: $DL > PL$



14. Given: ΔABC lies in plane M, with $\angle ABC \cong \angle ACB$. $m\angle PAC > m\angle PAB$.

Prove: $m\angle PBC > m\angle PCB$



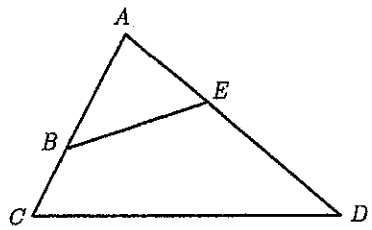
Chapter 3 Review Exercises

1. The sum of the measures of the interior angles of a polygon is twice the sum of the measures of the interior angles of an octagon. How many sides does the polygon have?

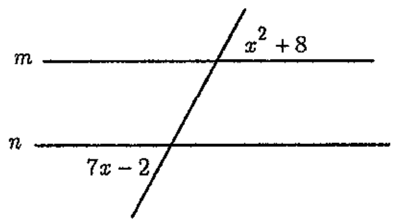
2. In a regular polygon, the ratio of the measure of an exterior angle to the measure of an interior angle is $1:4$. How many sides does the polygon have?

3. Given: $\angle D \cong \angle ABE$
   $\overline{BE} \nparallel \overline{CD}$

   Prove: $\overline{AC} \ncong \overline{AD}$



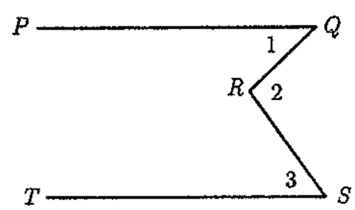
4. Find the value(s) of x that will make $m \parallel n$.



5. The measure of one angle of a triangle is 5 times as large as the measure of the smallest angle, and the remaining angle is 3 times as large as the smallest angle. Find the measure of each angle of the triangle.

6. a. The sum of the measures of the interior angles of a convex polygon is 900. How many sides does the polygon have?
   b. A regular polygon has exterior angles each of which has a measure of 45. How many sides does the polygon have?
   c. The sum of the measures of the interior angles of a convex polygon equals the sum of the measures of the exterior angles. How many sides does the polygon have?

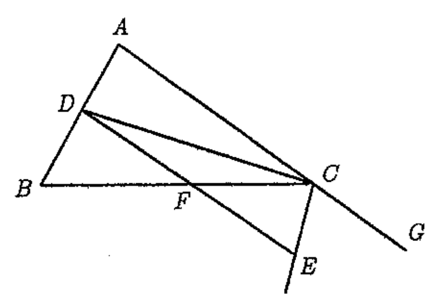
7. In the figure to the right, if $m\angle 1 = 36$, $m\angle 2 = 91$, and $m\angle 3 = 55$, then what can be said about $\overleftrightarrow{PQ}$ and $\overleftrightarrow{TS}$? Justify your answer.



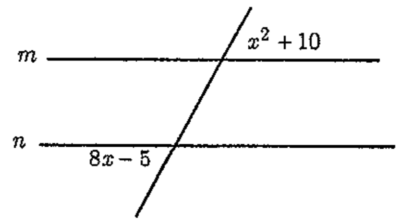
8. The bisector of $\angle B$ in ΔABC intersects $\overline{AC}$ at D. Through A a line is drawn parallel to $\overline{BD}$, intersecting $\overleftrightarrow{BC}$ at E. Prove that ΔEBA is isosceles.

9. Given: $\overline{CD}$ bisects $\angle ACB$;
   $\overline{CE}$ bisects $\angle BCG$;
   $\overline{DF} \parallel \overline{AC}$.

   Prove: F is equidistant from C, D, and E.



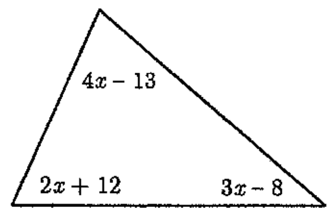
10. Find the value(s) of x that will make $m \parallel n$.



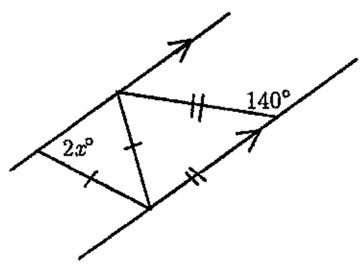
11. $\overline{AE}$ is the bisector of $\angle A$ in ΔABC. D is on $\overline{AC}$ such that $AD = DE$. Prove that $\overline{DE} \parallel \overline{AB}$.

12. Prove that if the bisector of an exterior angle of a triangle is parallel to a side of the triangle, then the triangle is isosceles.

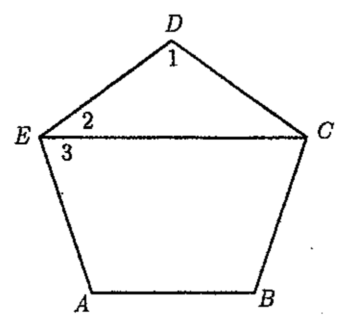
13. Determine the value of x in the figure to the right.



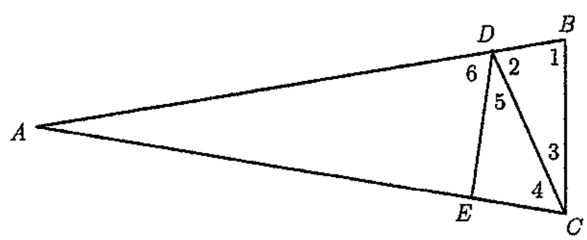
14. Determine x in the figure to the right.



15. Pictured to the right is **regular** pentagon $ABCDE$.
    a. Determine the measure of each of the numbered angles.
    b. Is $\overline{AB} \parallel \overline{EC}$? Explain briefly.



16. Given:
$\overline{AB} \cong \overline{AC}$
$\overline{BC} \cong \overline{CD}$
$\overline{DE} \perp \overline{AC}$
$m\angle A = 20$

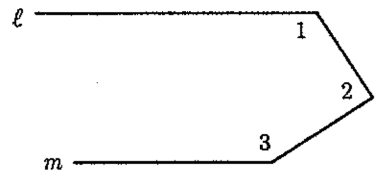


Determine the measure of each of the numbered angles.

17. ΔABC is a right triangle with a right angle at C. From C a perpendicular is drawn to $\overline{AB}$, meeting $\overline{AB}$ at D. From D a perpendicular is drawn to $\overline{BC}$ meeting $\overline{BC}$ at E. If $m\angle A = 48$, determine $m\angle ACD$, $m\angle DCE$, $m\angle CDE$, $m\angle BDE$, and $m\angle B$.

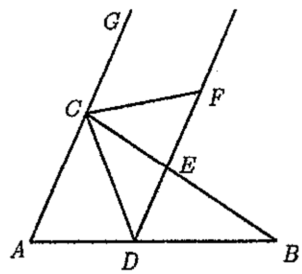
18. Given: $\ell \parallel m$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 360$



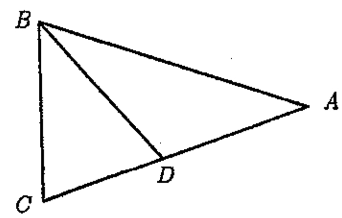
19. Given: $\overline{CD}$ bisects $\angle ACB$;
$\overline{DF} \parallel \overline{AC}$;
$\overline{CF}$ bisects $\angle GCB$.

Prove: E is the midpoint of $\overline{DF}$.

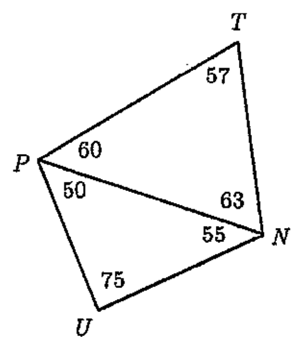


20. Given: ΔABC is isosceles with $AB = AC$.
$AD = BD = BC$.

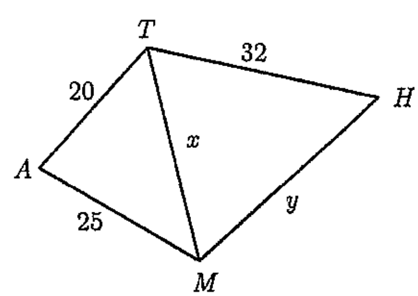
Prove: $m\angle A = 36$



21. Given the figure to the right with angle measures as indicated, arrange the lengths of the five segments in order of increasing length. [Note: The figure is **not** drawn to scale.]

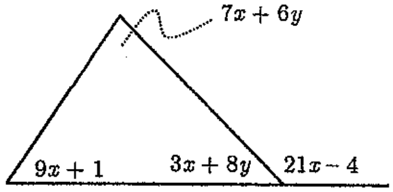


22. In the figure, the lengths of three sides are known.
   a. Determine the range of values that x can take on.
   b. Determine the range of values that y can take on.
   c. If x and y are integers, then what is the largest possible value for $x+y$?

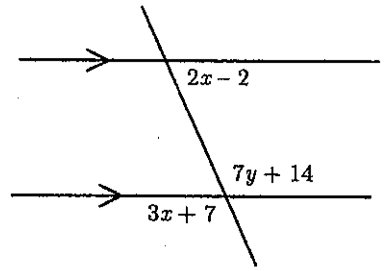


23. Using the angle measures indicated, determine both x and y in each of the following.

a.

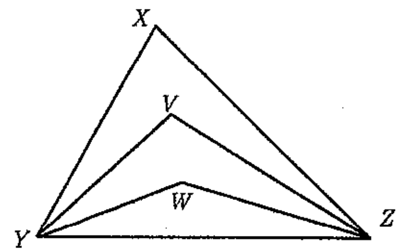


b.



24. Given: In ΔXYZ, $XZ > XY$, $\overline{YV}$ and $\overline{YW}$ trisect (divide into three congruent parts) $\angle XYZ$ while $\overline{ZV}$ and $\overline{ZW}$ trisect $\angle XZY$.

Prove: $WZ > WY$



25. In ΔDAP, R is the midpoint of $\overline{DP}$. If $\angle ARP$ is obtuse, prove that $m\angle D > m\angle P$.

26. Two sides of a regular nonagon (a nine-sided polygon) are extended as shown. What is the measure of the angle they form?