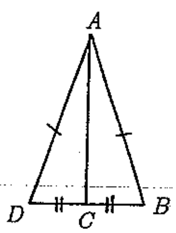
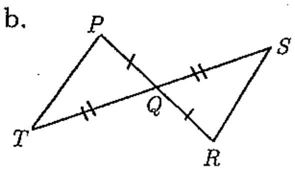
Exercises 2.1

1. Write an appropriate congruence between the two triangles, and identify the postulate used.

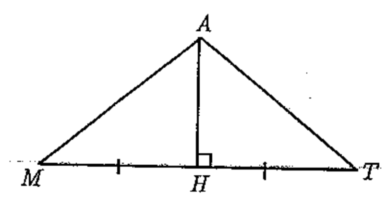
a.



b.

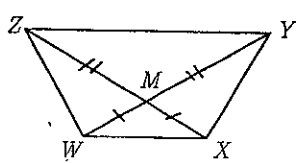


c.

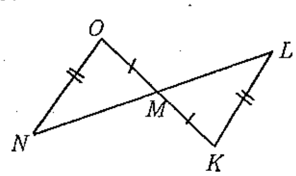


2. In each of the following, determine a congruence between two triangles if one exists. Identify the congruence and the appropriate postulate. If no congruence exists, say so.

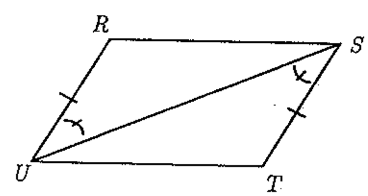
a.



b.

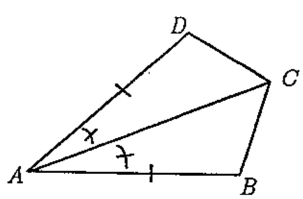


c.

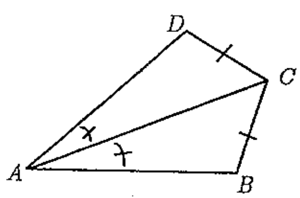


3. In each of the following, determine a congruence between two triangles if one exists. Identify the congruence and the appropriate postulate. If no congruence exists, say so.

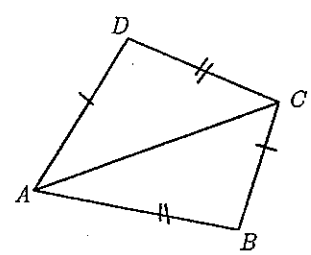
a.



b.

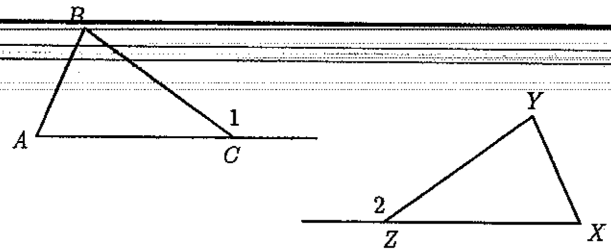


c.



4. Given: $\triangle ABC \cong \triangle XYZ$

Prove: $\angle 1 \cong \angle 2$

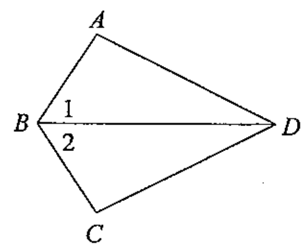


5. a. Draw a triangle with three acute angles. Now draw the three altitudes of the triangle.
   b. Draw a right triangle. Now draw the three altitudes of the triangle.
   c. Draw a triangle with an obtuse angle. Now draw the three altitudes.

6. a. Draw a triangle and draw its three medians.
   b. Draw a triangle and put in the bisectors of the three angles.

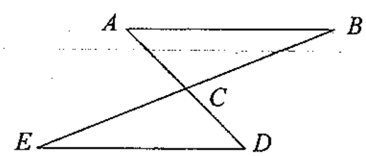
7. Given: $\overline{AB} \cong \overline{BC}$, $\overline{AD} \cong \overline{CD}$

Prove: $\angle 1 \cong \angle 2$



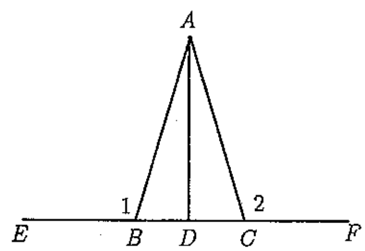
8. Given: C is the midpoint of $\overline{AD}$ and $\overline{BE}$.

Prove: $\angle B \cong \angle E$



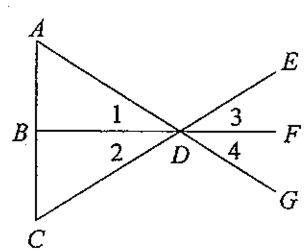
9. Given: $\overline{AD} \perp \overline{EF}$, $\overline{BD} \cong \overline{CD}$

Prove: $\angle 1 \cong \angle 2$



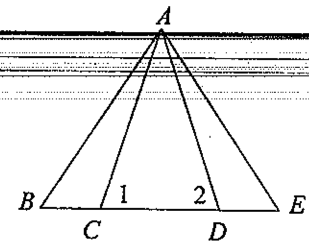
10. Given: $\triangle ABD \cong \triangle CBD$

Prove: $\overline{BF}$ bisects $\angle EDG$



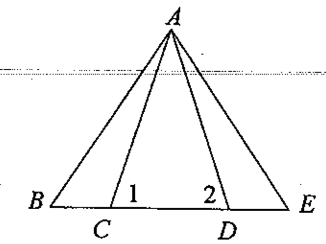
11. Given: $\overline{BC} \cong \overline{DE}$, $\overline{AB} \cong \overline{AE}$, $\overline{AC} \cong \overline{AD}$

Prove: $\angle 1 \cong \angle 2$



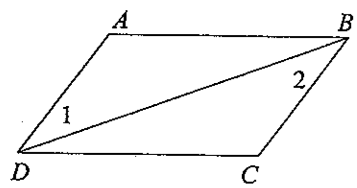
12. Given: $\overline{BC} \cong \overline{DE}$, $\overline{AB} \cong \overline{AE}$, $\overline{AC} \cong \overline{AD}$

Prove: $\angle BAD \cong \angle EAC$



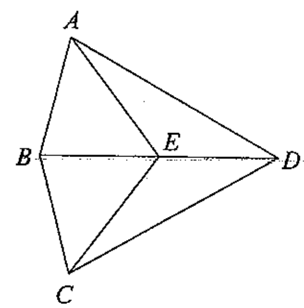
13. Given: $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$

Prove: $\angle 1 \cong \angle 2$



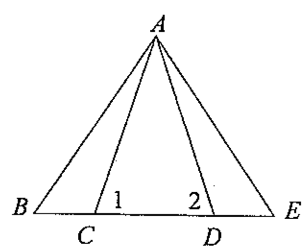
14. Given: $\Delta ABE \cong \Delta CBE$

Prove: $\Delta AED \cong \Delta CED$



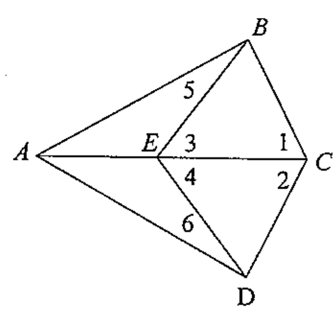
15. Given: $\angle 1 \cong \angle 2$, $\overline{BC} \cong \overline{DE}$, $\overline{AD} \cong \overline{AC}$

Prove: $\angle B \cong \angle E$



16. Given: $\angle 1 \cong \angle 2$, $\overline{BC} \cong \overline{DC}$

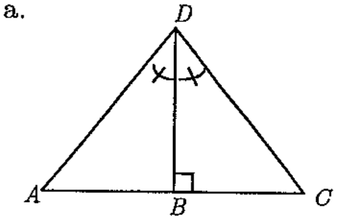
Prove: $\angle 5 \cong \angle 6$



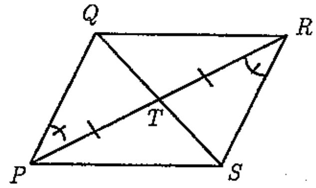
Exercises 2.2

1. Write an appropriate congruence between two triangles, and identify the postulate used.

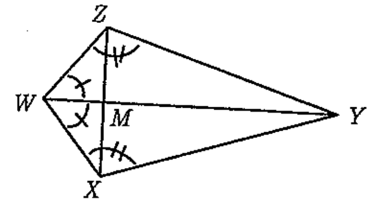
a.



b.

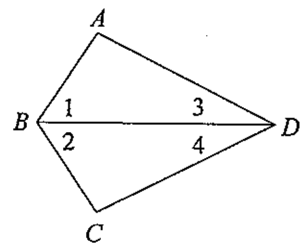


c.



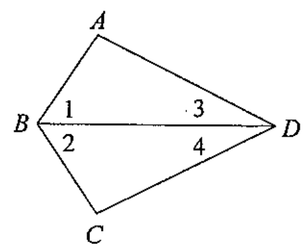
2. <u>Given</u>: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

<u>Prove</u>: $\overline{AD} \cong \overline{CD}$



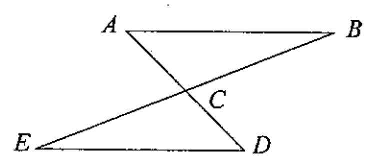
3. <u>Given</u>: $\angle 1 \cong \angle 2$, $\angle A \cong \angle C$

<u>Prove</u>: $\overline{BD}$ bisects $\angle ADC$



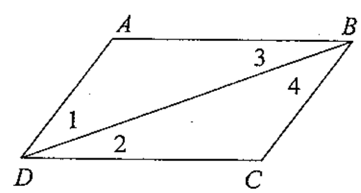
4. <u>Given</u>: $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$

<u>Prove</u>: $\overline{BC} \cong \overline{EC}$



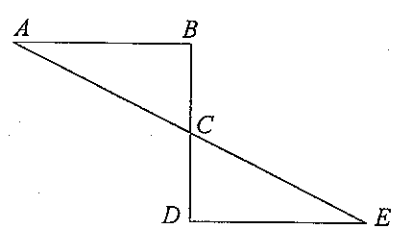
5. <u>Given</u>: $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$

<u>Prove</u>: $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$



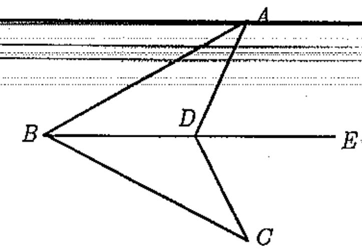
6. <u>Given</u>: $\overline{BD} \perp \overline{AB}$, $\overline{BD} \perp \overline{DE}$, C is the midpoint of $\overline{BD}$.

<u>Prove</u>: $\angle E \cong \angle A$



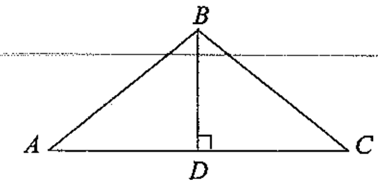
~~7. Given: $\overline{BE}$ bisects both $\angle ABC$ and $\angle ADC$.~~

Prove: $\overline{AD} \cong \overline{DC}$



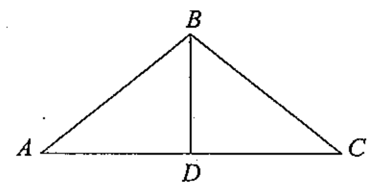
8. Given: $\overline{BD} \perp \overline{AC}$; $\overline{BD}$ bisects $\angle ABC$.

Prove: $\overline{AB} \cong \overline{BC}$



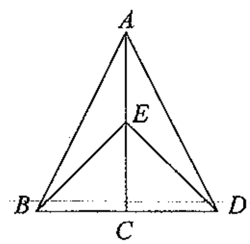
9. Given: $\angle A \cong \angle C$, $\overline{BD}$ bisects $\angle ABC$.

Prove: D is the midpoint of $\overline{AC}$.



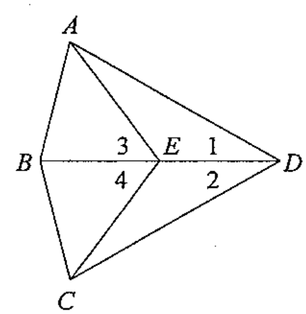
10. Given: $\overline{AB} \cong \overline{AD}$, $\overline{BE} \cong \overline{DE}$

Prove: $\angle EBC \cong \angle EDC$



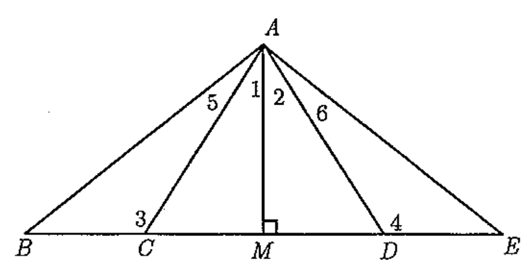
11. Given: $\angle 1 \cong \angle 2$, $\overline{AD} \cong \overline{CD}$

Prove: $\angle BAE \cong \angle BCE$



12. Given: $\angle 1 \cong \angle 2$, $\overline{AM} \perp \overline{BE}$, $\angle B \cong \angle E$

Prove: $\angle 5 \cong \angle 6$



Exercises 2.3

In problems 1–3, you are asked to prove a theorem. Prior to writing the proof, you must supply a diagram and restate the theorem in terms of the diagram.

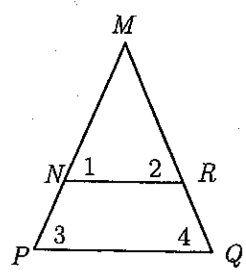
1. a. Prove **Corollary 2.3**: Every equilateral triangle is equiangular.
   b. Prove **Corollary 2.4**: Every equiangular triangle is equilateral.

2. Prove **Theorem 2.5**: The bisector of the vertex angle of an isosceles triangle is perpendicular to the base. Use Thm 1.7

3. Prove **Theorem 2.6**: The median drawn from the vertex angle of an isosceles triangle is the angle bisector of the vertex angle.

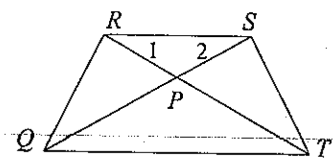
4. Given: $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$

   Prove: $NP = RQ$



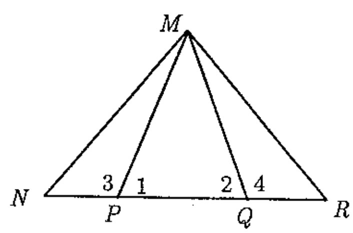
5. Given: $\overline{QS} \cong \overline{TR},\ \overline{QR} \cong \overline{TS}$

   Prove: $\overline{RP} \cong \overline{SP}$



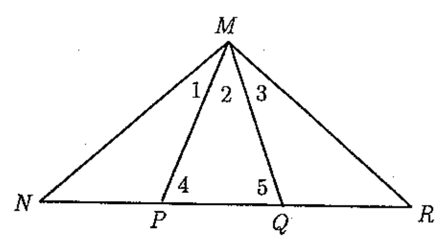
6. Given: $\overline{MP} \cong \overline{MQ},\ \overline{NP} \cong \overline{QR}$

   Prove: ΔMNR is isosceles.



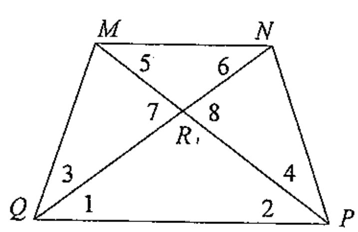
7. Given: $\overline{MN} \cong \overline{MR},\ \angle 1 \cong \angle 2 \cong \angle 3$

   Prove: $\angle 4 \cong \angle 5$



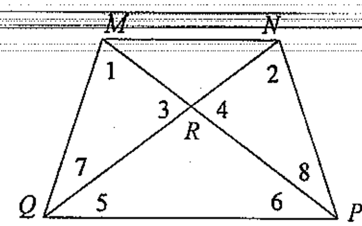
8. Given: $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$

   Prove: $\angle 5 \cong \angle 6$



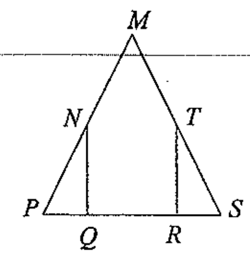
9. Given: $\angle 1 \cong \angle 2$, $\overline{MQ} \cong \overline{NP}$

Prove: $\Delta MQP \cong \Delta NPQ$



10. Given: Isosceles ΔMPS with $\overline{MP} \cong \overline{MS}$; N and T are midpoints of $\overline{MP}$ and $\overline{MS}$ respectively; $\overline{NQ} \perp \overline{PS}$, $\overline{TR} \perp \overline{PS}$.

Prove: $\overline{NQ} \cong \overline{TR}$



In problems 11–13, prior to proving the theorem, draw a diagram exemplifying the problem, rewrite the theorem in if-then form, and finally express the Given and Prove in terms of the diagram.

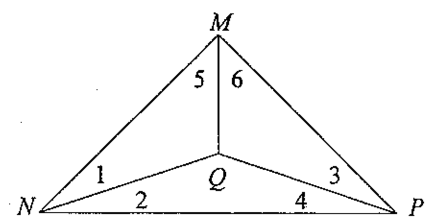
11. Prove **Theorem 2.7:** The medians drawn to the legs of an isosceles triangle are congruent.

12. Prove **Theorem 2.8:** The angle bisectors of the base angles of an isosceles triangle are congruent.

13. Prove **Theorem 2.9:** The altitudes drawn from the base angles of an isosceles triangle are congruent.

14. Given: $\overline{MN} \cong \overline{MP}$, $\overline{NQ}$ bisects $\angle MNP$, and $\overline{PQ}$ bisects $\angle MPN$.

Prove: $\overline{NQ} \cong \overline{PQ}$

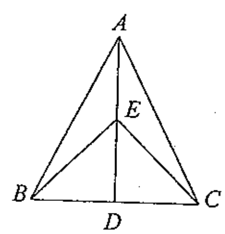


Exercises 2.4:

1. If $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$, prove that $\angle ABC \cong \angle ADC$.

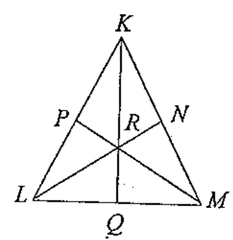
2. <u>Given</u>: $\triangle ABD \cong \triangle ACD$

   <u>Prove</u>: $\triangle BEC$ is isosceles



3. <u>Given</u>: $KL = KM$, $\overline{MP}$ and $\overline{LN}$ are medians.

   <u>Prove</u>: Line segment $\overline{KRQ}$ is the perpendicular bisector of $\overline{LM}$.



~~Problems 4–7 are theorems and require a diagram and Given and Prove in terms of the diagram. Only #5 is a theorem that we will use in other proofs.~~

4. Prove **Theorem 2.14**: In a plane, if each of two points is equidistant from the endpoints of a segment, then the line they lie on is the perpendicular bisector of the segment.

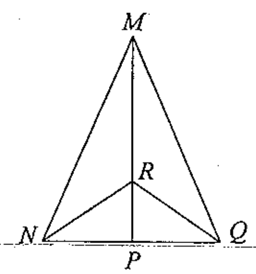
5. ~~Prove: If the median to the hypotenuse of a right triangle is perpendicular to the~~ hypotenuse, then the right triangle is isosceles.

6. Prove: If one diagonal of a quadrilateral bisects two angles of the quadrilateral, then it bisects the other diagonal.

7. Prove: If P is not equidistant from the endpoints of segment $\overline{AB}$, then P is not on the perpendicular bisector of $\overline{AB}$.

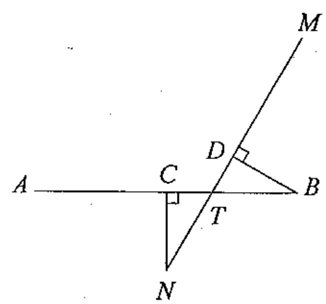
8. Given: $MN = MQ$, $\overline{RN}$ bisects $\angle MNQ$, and $\overline{RQ}$ bisects $\angle MQN$.

   Prove: $\overline{MP}$ is the perpendicular bisector of $\overline{NQ}$



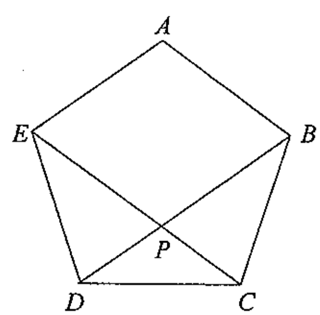
9. Given: $\overline{NC}$ and $\overline{BD}$ are the perpendicular bisectors of $\overline{AB}$ and $\overline{NM}$ respectively.

   Prove: $AN = BM$



10. Given: $\angle ABC \cong \angle BCD \cong \angle CDE \cong \angle DEA \cong \angle EAB$; $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EA}$

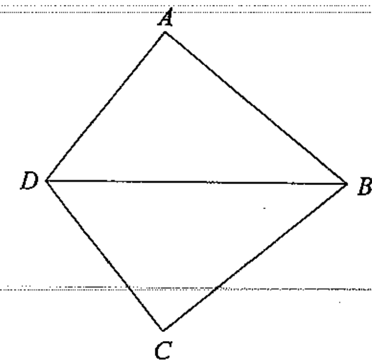
    Prove: P lies on the perpendicular bisector of $\overline{DC}$.



Exercises 2.5:

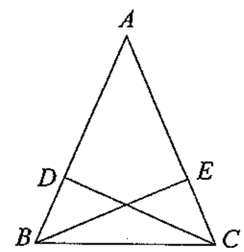
1. Given: $\overline{AD} \cong \overline{DC}$, $\overline{DA} \perp \overline{AB}$, $\overline{DC} \perp \overline{BC}$

   Prove: $\overline{DB}$ bisects $\angle ABC$



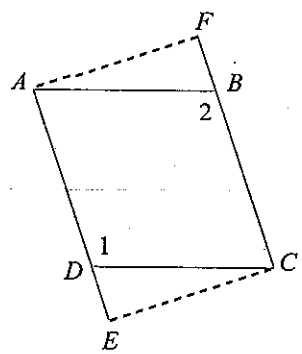
2. Given: $\overline{CD} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$, $\overline{DB} \cong \overline{EC}$

   Prove: ΔABC is isosceles.



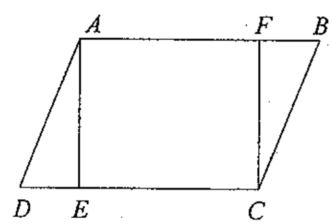
3. Given: $\overline{AB} \cong \overline{DC}$, $\overline{AF} \cong \overline{CE}$, $\overline{AF} \perp \overline{FC}$, and $\overline{CE} \perp \overline{AE}$.

   Prove: $\angle 1 \cong \angle 2$



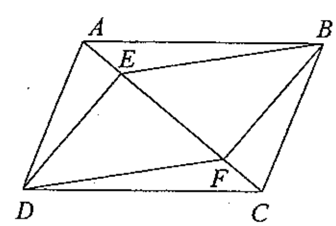
4. Given: $\overline{AD} \cong \overline{BC}$, $\overline{DE} \cong \overline{BF}$, $\overline{AE} \perp \overline{DC}$, and $\overline{CF} \perp \overline{AB}$.

   Prove: $\overline{AE} \cong \overline{CF}$, $\angle D \cong \angle B$

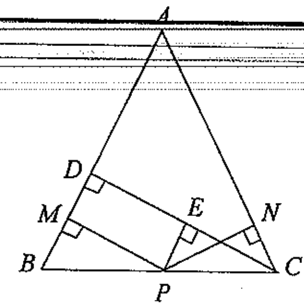


5. Given: $\overline{AD} \cong \overline{BC}$, $\overline{AE} \cong \overline{FC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{BF} \perp \overline{AC}$.

   Prove: $\overline{EB} \cong \overline{DF}$



6. The perpendiculars are as shown in the diagram. Also, $PE \cong NC$ and $DE \cong MP$. Prove that $MP + NP = DC$. Use a paragraph proof.

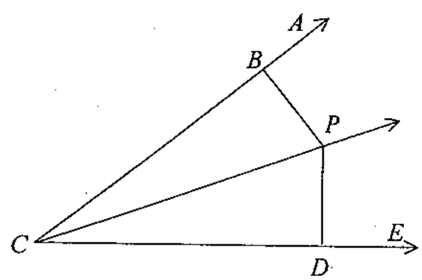


Problems 7 and 8 constitute the proof of the Angle Bisector Characterization Theorem. The diagram, the Given and Prove are present. In the other problems you must express the theorem with a diagram and restate the Given and Prove in terms of the diagram before writing the proof.

7. Prove the first part of **Theorem 2-16:** If a point lies on the bisector of an angle, it is equidistant from the sides of the angle.

Given: $\overline{CP}$ bisects $\angle ACE$, $\overline{PB} \perp \overline{AC}$, and $\overline{PD} \perp \overline{CE}$.

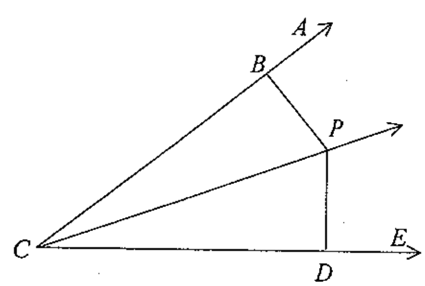
Prove: $PB = PD$



8. Prove the second part of **Theorem 2-16:** In a plane, if a point is equidistant from the sides of an angle, it lies on the bisector of the angle.

Given: $\overline{PB} \perp \overline{AC}$, $\overline{PD} \perp \overline{CE}$, and $PB = PD$

Prove: $\overline{CP}$ bisects $\angle ACE$



9. Given two adjacent supplementary angles, prove that their bisectors are perpendicular.

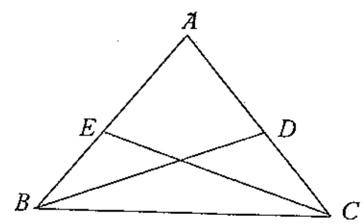
10. Prove: If two altitudes of a triangle are congruent, then the triangle is isosceles.

11. Prove: The altitude to the base of an isosceles triangle is the median to the base and the angle bisector of the vertex angle of the triangle.

Exercises 2.6:

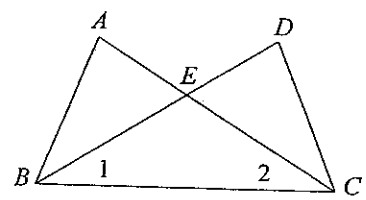
1. Given: $\overline{BE} \cong \overline{CD}$, $\overline{BD} \cong \overline{CE}$

   Prove: ΔABC is isosceles.



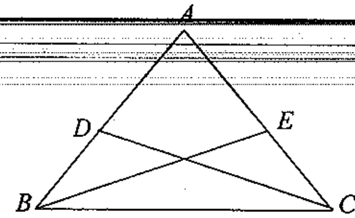
2. Given: $\angle 1 \cong \angle 2$, $\angle A \cong \angle D$

   Prove: $\overline{AB} \cong \overline{DC}$



3. Given: $\overline{AB} \cong \overline{AC}$, $\angle BDC \cong \angle CEB$

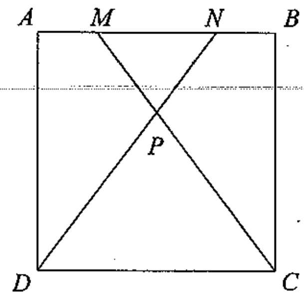
Prove: $\overline{CD} \cong \overline{BE}$



4. Given: Square $ABCD$, $AM = NB$

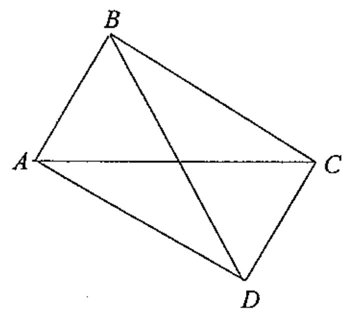
Prove: $\overline{MP} \cong \overline{NP}$

Write a paragraph proof.



5. Given: $\overline{AB} \perp \overline{AD}$, $\overline{AB} \perp \overline{BC}$, $\overline{AC} \cong \overline{BD}$

Prove: $\overline{AD} \cong \overline{BC}$



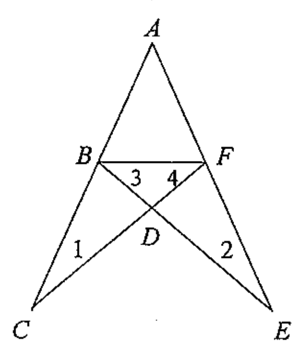
6. In pentagon $ABCDE$ all sides are equal and all angles are congruent. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at P. Using a paragraph proof, prove that $\overline{AC} \cong \overline{BD}$ and that ΔBPC is isosceles.

7. In hexagon $ABCDEF$ all sides are equal and all angles are congruent. Diagonals, $\overline{AC}$ and $\overline{BD}$ intersect at P. Using a paragraph proof, prove that $\overline{AC} \cong \overline{BD}$ and that ΔBPC is isosceles.

8. Generalize problems 6 and 7 to a polygon with n sides (where $n \geq 4$). Prove your generalization.

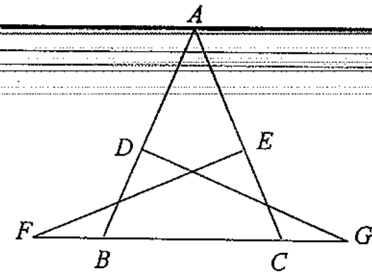
9. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: ΔABF is isosceles.



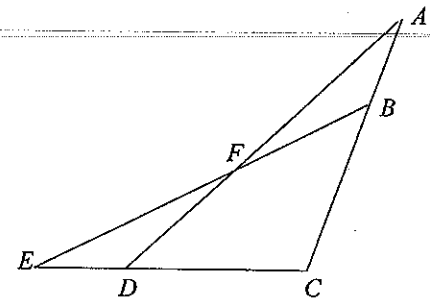
10. Given: $\overline{DB} \cong \overline{EC}$, $FB = GC$, $\overline{GD} \perp \overline{AB}$, and $\overline{FE} \perp \overline{AC}$.

Prove: $\overline{AB} \cong \overline{AC}$



11. Given: $AB = ED$, $BC = DC$

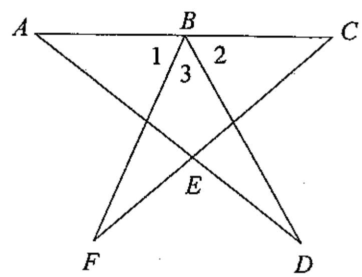
Prove: $\overline{AF} \cong \overline{EF}$



12. Given: $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{CB}$, $\overline{BF} \cong \overline{BD}$

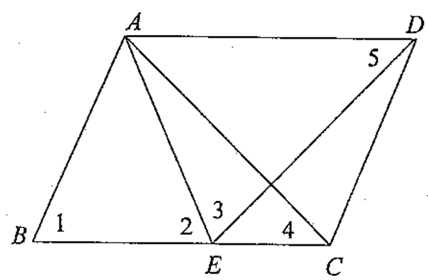
Prove: $\overline{AD} \cong \overline{CF}$

Then continue by proving that ΔACE is isosceles using a paragraph proof.



13. Given: $\angle 1 \cong \angle 2 \cong \angle 3$, $\overline{BC} \cong \overline{ED}$

Prove: $\angle 4 \cong \angle 5$

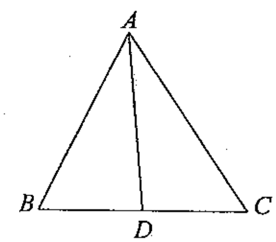


Exercises 2.7

1. Determine what is wrong with the following proof that all triangles are isosceles.

Given: ΔABC

Prove: ΔABC is isosceles.

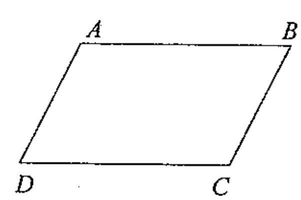


Given ΔABC, let D be the midpoint of $\overline{BC}$ and draw the perpendicular to $\overline{BC}$ through D. Since $BD = DC$ because D is the midpoint, $AD = AD$ by the reflexive property and $\angle ADB \cong \angle ADC$ since both are right angles, $\Delta ABD \cong \Delta ACD$ by SAS. By CPCTC, $AB = AC$, making ΔABC isosceles.

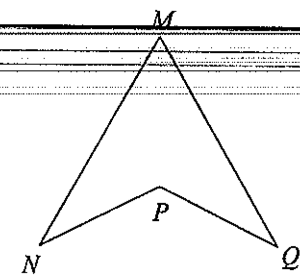
2. Using the diagram in #1, here's another proof that all triangles are isosceles. Find what's wrong.

Draw the angle bisector $\overline{AD}$ of $\angle BAC$ perpendicular to $\overline{BC}$. Since $\angle BAD \cong \angle CAD$, $AD = AD$, and $\angle ADB \cong \angle ADC$, then $\Delta ABD \cong \Delta ACD$ by ASA. By CPCTC, $AB = AC$, making ΔABC isosceles.

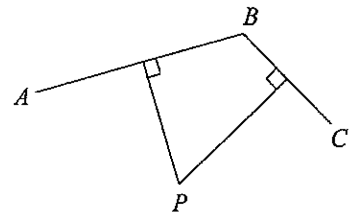
3. If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, prove that $\angle B \cong \angle D$



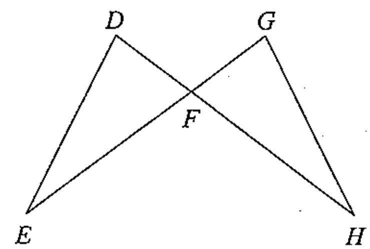
4. If $\overline{MN} \cong \overline{MQ}$ and $\overline{NP} \cong \overline{QP}$, prove that $\angle N \cong \angle Q$.



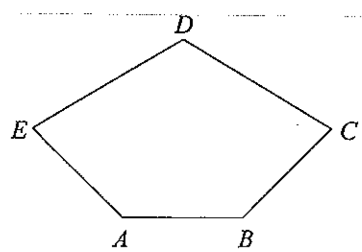
5. P lies on the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ respectively. Prove that $PA = PC$.



6. If $\overline{DE} \cong \overline{GH}$ and $\overline{EG} \cong \overline{DH}$, prove that $\angle E \cong \angle H$.

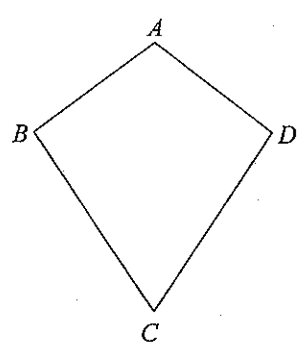


7. In the figure $\overline{AE} \cong \overline{BC}$, $\overline{DE} \cong \overline{DC}$, and $\angle E \cong \angle C$. Prove that D lies on the perpendicular bisector of $\overline{AB}$.

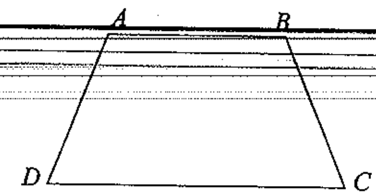


8. <u>Given</u>: $\overline{AB} \cong \overline{AD}$ and $\angle B \cong \angle D$,

<u>Prove</u>: $\overline{BC} \cong \overline{DC}$

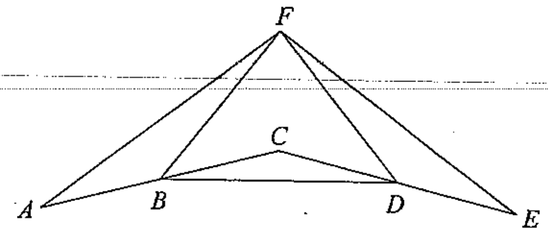


~~9. If $\overline{AD} \cong \overline{BC}$ and $\angle D \cong \angle C$, prove that $\angle A \cong \angle B$.~~



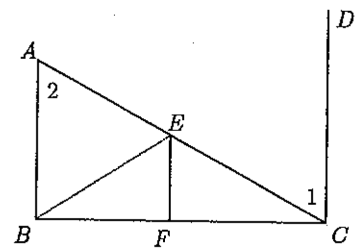
10. <u>Given</u>: $\overline{AF} \cong \overline{EF}$, $\overline{AC} \cong \overline{EC}$, and ~~$\angle AFB \cong \angle EFD$~~

<u>Prove</u>: ΔBDF is isosceles



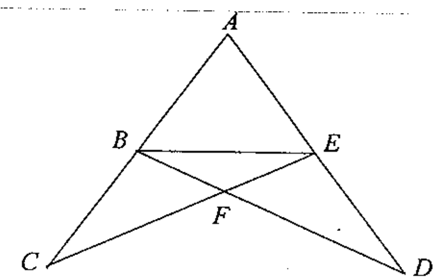
11. <u>Given</u>: $\overline{AB}$, $\overline{DC}$, and $\overline{EF}$ are perpendicular to $\overline{BC}$, F is the midpoint of $\overline{BC}$, and $\angle 1 \cong \angle 2$.

<u>Prove</u>: $AE = EC = BE$

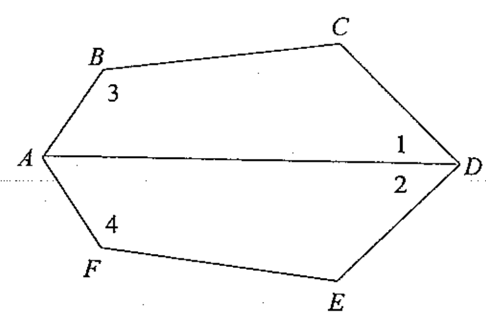


12. <u>Given</u>: $\overline{AC} \cong \overline{AD}$ and $\overline{CF} \cong \overline{DF}$

<u>Prove</u>: $\overline{BF} \cong \overline{EF}$

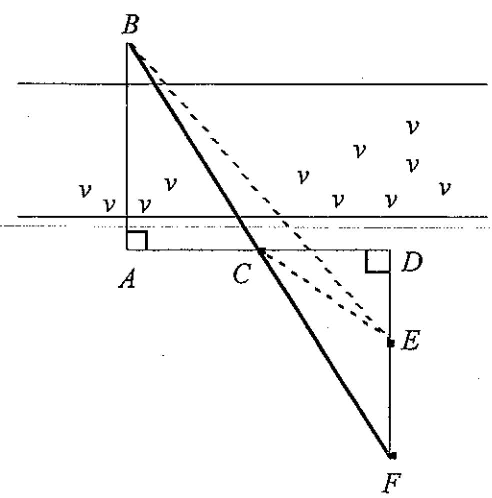


13. Given $\overline{AB} \cong \overline{AF}$, $\overline{BC} \cong \overline{FE}$, $\overline{CD} \cong \overline{ED}$, and $\angle 1 \cong \angle 2$, prove that $\angle 3 \cong \angle 4$.

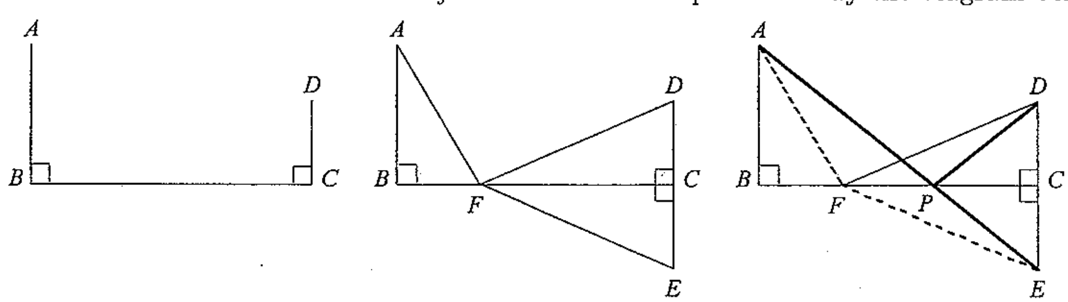


Applications

1. Imagine that you must cross a river of overly eager velociraptors. See the diagram below and feel their hunger. You'd obviously like to topple a tree to provide a bridge but you need to know how wide the river is and you certainly can't measure it directly. So you use your noggin and your knowledge of mathematics and do either of the following:
   a. Stand upright near the bank and sight along the brim of your hat to a point just on the other side. Keeping your body rigid, rotate until sighting along the brim of your hat brings you to a point on the ground on your side. Pace off the distance to that point and that is the distance across the river. Explain why this method works.
   b. Standing near the bank at A, sight at a small shrub B on the other side. Turn 90° and walk a certain distance, say 10 yards, parallel to the bank, and mark your stopping point C. Then continue walking in the same direction the same distance to D. Turn 90° again and walk. You'll notice that there is an angle formed by you, point C and point B. That angle is diminishing until B and C are collinear. Stop when that occurs. The distance you've walked perpendicular to the river is the distance across the river. Why?



2. Suppose you're curious about the behavior of light. More specifically, you want to know the path taken by light that starts at point A and bounces off a mirror represented by line segment $\overline{BC}$, and ends up at point D as shown in the leftmost diagram. Since light seems to travel pretty fast, you assume that light will take the shortest path from A to the mirror and thence to D. How do you determine that path? Study the diagram below.

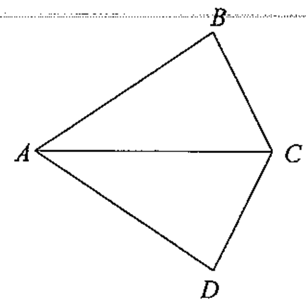


Consider reflecting point D across $\overline{BC}$ to position E as shown below in the middle diagram. Prove that the path $AF + FD$ has the same length as $AF + FE$. Now, as shown in the third diagram, draw a line from A to E. Prove that $AP + PD = AP + PE$. This is clearly the shortest path. Now prove that $\angle APB \cong \angle DPC$. You've essentially proved that the angle of incidence equals the angle of reflection.

Numerical problems

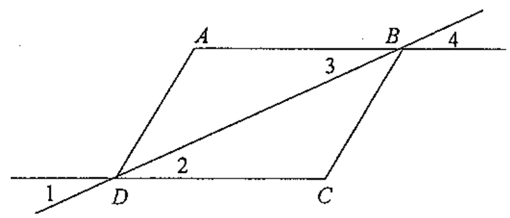
3. a. If $AB = 3x - 4$, $BC = x + 3$, $AD = 2x + 6$, and $DC = 2x - 2$, is $\Delta ABC \cong \Delta ADC$?

   b. If $AB = 3x - 2$, $BC = x + 3$, $AD = 2x + 3$, and $DC = 2x - 2$, is $\Delta ABC \cong \Delta ADC$?



4. a. If $m\angle 1 = 10x - 7$, $m\angle 4 = 8x + 5$, $AB = 7x + 4$, and $DC = x^2 + 10$, is $\Delta ABD \cong \Delta CDB$?

   b. If $m\angle 1 = 3x + 2y$, $m\angle 4 = 2x + 3y - 2$, $AB = 6x + 7y$, and $DC = x + 12y$, is $\Delta ABD \cong \Delta CDB$?

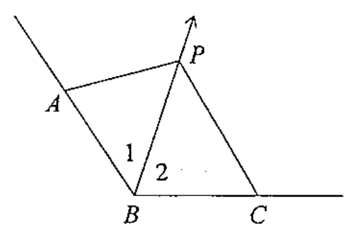


Incorrect Use of Auxiliary Lines

5. Regretfully, we made a mistake in the following proof. Find it.

Given: $\overline{AB} \cong \overline{CB}$, P is any point in the interior of $\angle BAC$.

Prove: $\overline{PA} \cong \overline{PC}$

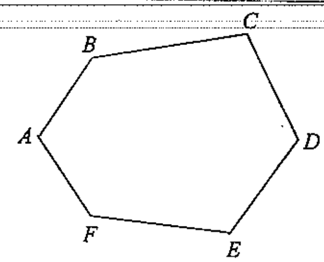


Draw the bisector of $\angle ABC$ passing through P. Since $\overline{AB} \cong \overline{CB}$, $\angle 1 \cong \angle 2$, and $\overline{BP} \cong \overline{BP}$, then $\Delta PAB \cong \Delta PCB$ by SAS making $\overline{PA} \cong \overline{PC}$.

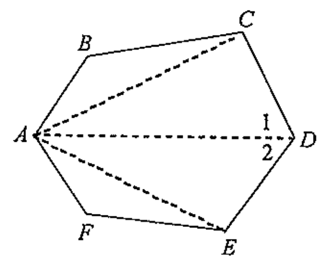
6. Ooops!

Given: $\overline{CD} \cong \overline{ED}$

Prove: $\angle CDA \cong \angle EDA$



Given figure $ABCDEF$, draw $\overline{AD}$ and then draw $\overline{AC}$ and $\overline{AE}$ perpendicular to $\overline{CD}$ and $\overline{ED}$ respectively. Since ΔACD and ΔAED are both right triangles with $\overline{CD} \cong \overline{ED}$ and $\overline{AD} \cong \overline{AD}$, then $\Delta ACD \cong \Delta AED$ by HL, giving $\angle 1 \cong \angle 2$.



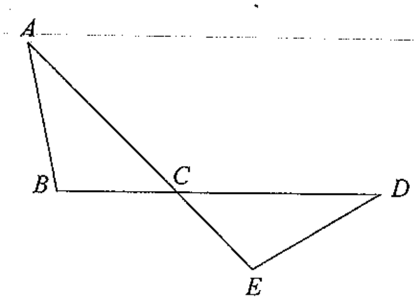
If you believe this conclusion, link some cardboard sticks together with the sticks of the same length representing $\overline{CD}$ and $\overline{ED}$, and see if those two angles are necessarily equal.

Auxiliary Line Problems

7. Given: $\overline{AB} \cong \overline{DE}$, $\overline{AE} \cong \overline{DB}$

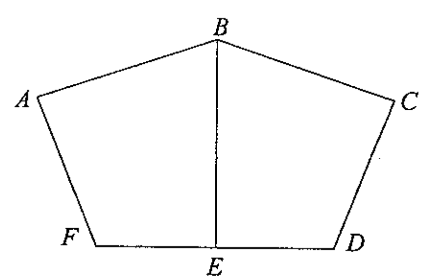
Prove: a. $\angle B \cong \angle E$

b. $\angle BAC \cong \angle EDC$



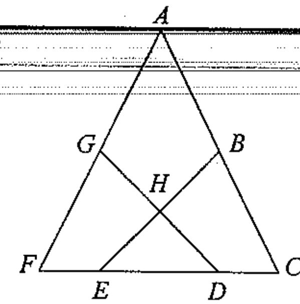
8. Given: $\overline{AB} \cong \overline{CB}$, $\overline{AF} \cong \overline{CD}$, $\angle A \cong \angle C$, and E is the midpoint of $\overline{FD}$.

Prove: $\overline{BE} \perp \overline{FD}$.



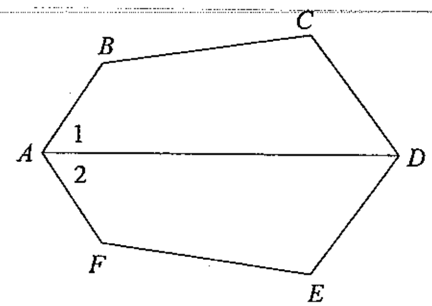
9. Given: $\angle F \cong \angle C$, $\overline{GH} \cong \overline{BH}$, G and B are midpoints of $\overline{AF}$ and $\overline{AC}$ respectively.

Prove: ΔHED is isosceles.



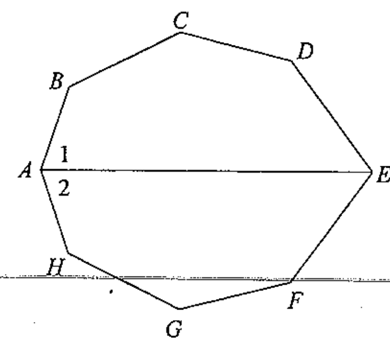
10. Given: $\overline{AB} \cong \overline{AF}$, $\overline{BC} \cong \overline{FE}$, $\angle 1 \cong \angle 2$, and $\angle B \cong \angle F$

Prove: $\overline{CD} \cong \overline{ED}$



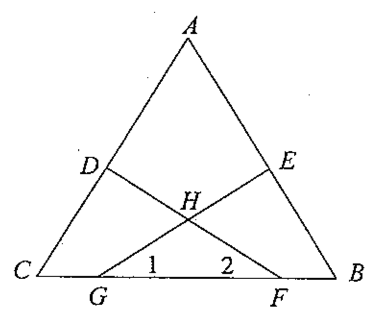
11. Given: $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{AH}$, $\overline{BC} \cong \overline{HG}$, $\overline{CD} \cong \overline{GF}$, $\overline{DE} \cong \overline{FE}$, and $\angle C \cong \angle G$

Prove: $\angle DEA \cong \angle FEA$



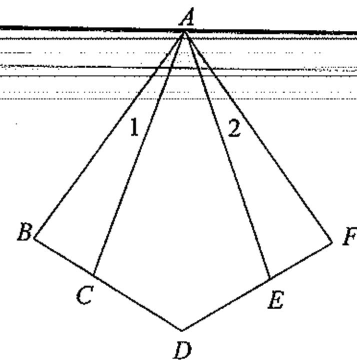
12. Given: $\overline{AD} \cong \overline{AE}$, $\angle 1 \cong \angle 2$, $\overline{FD} \perp \overline{AC}$, $\overline{GE} \perp \overline{AB}$.

Prove: ΔABC is isosceles.



13. Given: $\overline{AB} \cong \overline{AF}$, $\overline{BD} \cong \overline{FD}$, $\angle 1 \cong \angle 2$

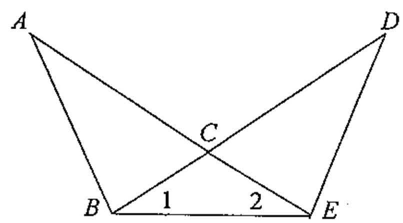
Prove: $\overline{CD} \cong \overline{ED}$.



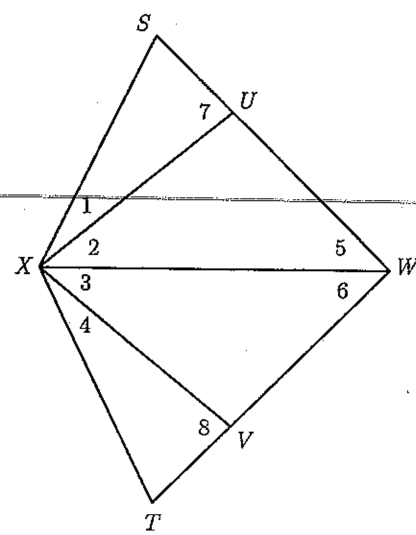
More sustained problems

14. Given: $\angle A \cong \angle D$; $\overline{AC} \cong \overline{DC}$.

Prove: $\angle 1 \cong \angle 2$

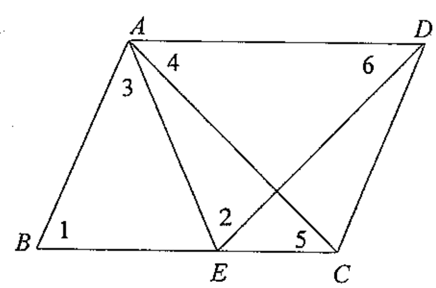


15. In the figure $\overline{XU} \cong \overline{XV}$, and $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$. Write a paragraph proof to show that $\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$.



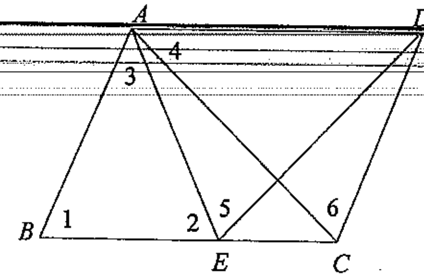
16. Given: $\overline{AB} \cong \overline{AE}$, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: $\angle 5 \cong \angle 6$

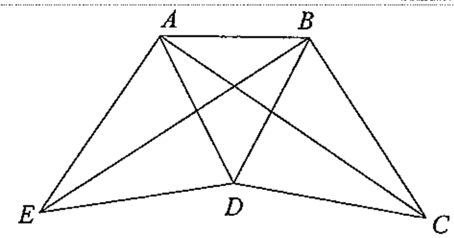


17. Given: $\overline{AC} \cong \overline{AD}$, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

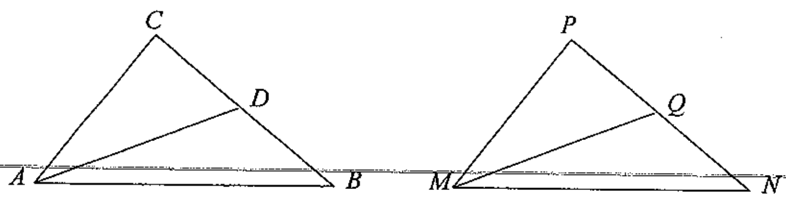
Prove: $\overline{EA}$ bisects $\angle BED$



18. In the figure, $\angle BAE \cong \angle ABC$, $\overline{AE} \cong \overline{BC}$, $\overline{DE} \cong \overline{DC}$, and $\overline{BD} \cong \overline{AD}$. Use a paragraph proof to show that $\Delta ADC \cong \Delta BDE$.



19. If $\overline{AC} \cong \overline{MP}$, $\overline{BC} \cong \overline{NP}$, and median $\overline{AD}$ is congruent to median $\overline{MQ}$, prove that $\Delta ABC \cong \Delta MNP$ using a paragraph proof.



20. Using the diagram in problem 19, modify the Given as follows and see what that does to the proof.
Write a paragraph proof to show that if $\overline{AC} \cong \overline{MP}$, $\overline{AB} \cong \overline{MN}$, and median $\overline{AD}$ is congruent to median $\overline{MQ}$, then $\Delta ABC \cong \Delta MNP$.