Exercises 1.1

1. Draw two lines m and k intersecting in a plane.

2. Draw lines m and k intersecting such that m lies in plane M and k does not.

3. Draw three parallel planes.

4. Draw two parallel planes M and N intersecting plane Q.

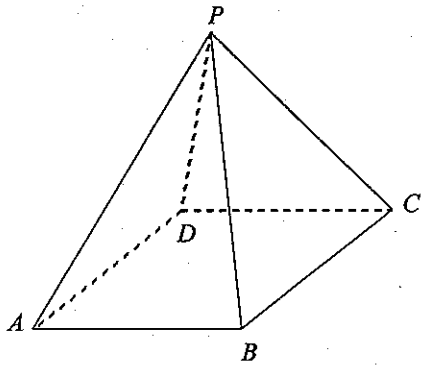
5. Draw two intersecting planes M and N such that line k intersects N but not M.

6. Draw lines m and k parallel to plane Q.

7. Draw two intersecting planes M and N such that line k lies in N but not M.

8. Draw three mutually perpendicular planes.

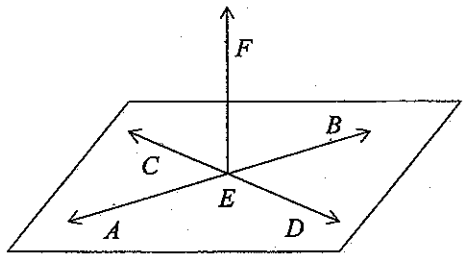
9. Given the pyramid $P\text{-}ABCD$, determine
   a. the intersection of planes PBC and PAB.
   b. the intersection of planes DBC and PAB.
   c. the intersection of planes PDA and PAB.



10. a. Are points C, F, E, and D coplanar?

    b. Are points F, B, and D coplanar?

    c. List all sets of 4 coplanar points.



11. Draw two planes that intersect at a single point.

Exercises 1.2

1. Make a diagram to illustrate each of the following situations.
   a. $\overline{AB}$ and $\overline{CD}$ bisect each other.
   b. $\overline{AB}$ bisects $\overline{CD}$, but $\overline{CD}$ doesn't bisect $\overline{AB}$.
   c. $\overline{AB}$ and $\overline{CD}$ intersect, but neither bisects the other.
   d. $\overline{AB}$ and $\overline{CD}$ do not intersect, but $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ do.

2. Point A lies on the number line.
   a. If the coordinate of A is 1, determine the number of points whose coordinate is an integer and that lie within 11.5 units of A.
   b. If the coordinate of A is 1.25, determine the number of points whose coordinate is an integer and that lie within 11.8 units of A.

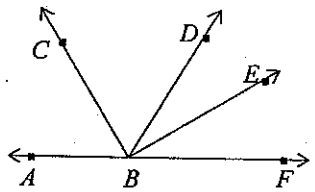
3. Let B be the midpoint of $\overline{AC}$. If $AB = 2x + 7$ and $AC = 54$, find x.

4. Let B be the midpoint of $\overline{AC}$. If $AB = 5x - 8$ and $BC = 3x + 6$, find x.

5. Let C be the midpoint of $\overline{AB}$, D be the midpoint of $\overline{AC}$ and E be the midpoint of $\overline{DC}$. If $AE = 16$, find DB.

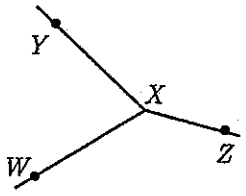
6. Points A, B, and C are collinear and B lies between A and C. If $AB = x^2 - 3$, $BC = 5x - 3$, and $AC = 18$, is B the midpoint of $\overline{AC}$? Why or why not?

7. Determine the number of angles in the figure to the right that are adjacent to $\angle CBD$.



8. Draw three angles $\angle ABC$, $\angle CBD$, and $\angle ABD$ for which it is not true that $m\angle ABC + m\angle CBD = m\angle ABD$.

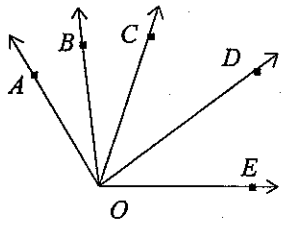
9. Name three pairs of adjacent angles in the figure to the right.



10. If you repeatedly bisect a 90° angle, you eventually arrive at an angle with a measure less than 1. How many times must you do the bisecting? What is the measure (expressed as a fraction rather than a decimal) of the resulting angle?

11. a. Using the diagram to the right, $m\angle AOE = 120$, $m\angle AOB = 18$, $\overline{OB}$ bisects $\angle AOC$, and $\overline{OD}$ bisects $\angle COE$. Find $m\angle COD$.

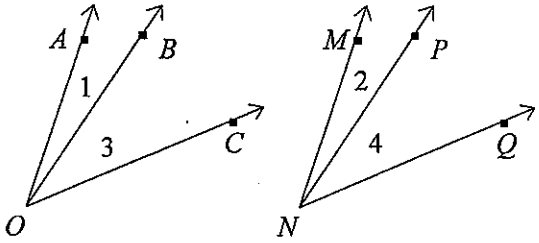
b. We have the same situation, except that $m\angle AOB = x$ and $m\angle AOE = 4x - 8$. Find $m\angle COD$ in terms of x.



12. Give the reasons for the following proof:

Given: $m\angle 1 = m\angle 2,\ m\angle 3 = m\angle 4$

Prove: $m\angle AOC = m\angle MNQ$

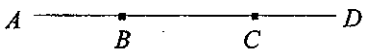


Statements	Reasons
1. $m\angle 1 = m\angle 2,\ m\angle 3 = m\angle 4$	1.
2. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	2.
3. $m\angle 1 + m\angle 3 = m\angle AOC,\ m\angle 2 + m\angle 4 = m\angle MNQ$	3.
4. $m\angle AOC = m\angle MNQ$	4.

13. Give the reasons for the following proof:

Given: $AB = CD$

Prove: $AC = BD$



Statements	Reasons
1. $AB = CD$	1.
2. $BC = BC$	2.
3. $AB + BC = BC + CD$	3.
4. $AB + BC = AC,\ BC + CD = BD$	4.
5. $AC = BD$	5.

In problems 14–17, B is the midpoint of $\overline{AC}$.

14. If $AB = x^2 + 3x$ and $BC = 8x - 4$, find all possible lengths of $\overline{AC}$.

15. If $AB = 2x^2 - x$ and $BC = 12 - 3x$, find all possible lengths of $\overline{AC}$.

16. Find all ordered pairs of positive integers (x, y) such that $AB = 2x + 1$ and $BC = 6 - y$.

17. D is on $\overline{BC}$, $AD = 13 - x$, and $DC = 3x + 1$. Determine the range of possible values for x.

Exercises 1.3

1. Write the negation or opposite of each statement.
   a. $x + 2 = 9$.
   b. The sum of two even numbers is even.

2. Many statements can be expressed in if-then form. For example: "all roses are flowers" could be expressed as "if a plant is a rose, then it is a flower." Rewrite the following as if-then statements.
   a. All rectangles are parallelograms.
   b. All circles are round.
   c. The sum of two odd numbers is even.
   d. The diagonals of a rectangle are congruent.

3. Draw a diagram to illustrate by a counterexample that the if-then statement is false.
   a. If a figure is a quadrilateral, then each pair of opposite sides is parallel.
   b. If a figure is a parallelogram, then a diagonal bisects the opposite angles.
   c. If a figure is a rhombus, then its diagonals are equal.
   d. If a figure is a kite, then each pair of opposite sides is parallel.

4. For the following statements write the converse and the contrapositive, then state whether the statement, converse, and contrapositive are true or not. If not true, give an example.

   a. If $x > 2^{64}$, then x is positive.
   b. If $x+4=7$, then $x=3$.
   c. If $x \geq 3$, then $x^2 \geq 9$.
   d. If the side of a square is 3, then the area of the square is 9.
   e. If the diagonals of a figure are perpendicular, then the figure is a square.

5. Write each pair of statements in "if and only if" form.

   a. If $2x-1=5$, then $x=3$.
      If $x=3$, then $2x-1=5$.
   b. If a triangle is isosceles, then it has two equal sides.
      If a triangle has two equal sides, then it is isosceles.
   c. If a figure has three sides, then the sum of its angles is 180.
      If the sum of the angles of a figure is 180, then it is a triangle.
   d. If all four sides of a figure are equal, then the figure is a rhombus.
      If a figure is a rhombus, then all four sides are equal.

In problems 6–9, let n, m, and k be positive integers. Turn each equation into a mathematical statement expressed in if-then form. For example, given $(2m)(2n)=4mn$ you could express that as "the product of two even numbers is even." You could also have said that "the product of two even numbers is a multiple of 4." In if-then form these become:

> If two even numbers are multiplied together, then the result is even.
> If two even numbers are multiplied together, then the result has 4 as a factor.
> If two even numbers are multiplied together, then the result is divisible by 4.

6. $(2m+1)(2n+1)=4mn+2m+2n+1=2(2mn+m+n)+1$

7. $(n-1)(n)(n+1)=6k$

8. $n^3+(n+1)^3+(n+2)^3=9k$

9. $1^3+2^3+3^3+\ldots+n^3=k^2$

10. Extend this table to confirm the pattern and then express the relationship in if-then form.

$$\begin{aligned} 3\cdot 37 &= 111 \quad \text{and} \quad 1+1+1=3 \\ 6\cdot 37 &= 222 \quad \text{and} \quad 2+2+2=6 \\ 9\cdot 37 &= 333 \quad \text{and} \quad 3+3+3=9 \\ 12\cdot 37 &= 444 \quad \text{and} \quad 4+4+4=12 \end{aligned}$$

11. Extend this table to confirm the pattern and then express the relationship in if-then form.

$$7 \cdot 15873 = 111111$$
$$14 \cdot 15873 = 222222$$
$$21 \cdot 15873 = 333333$$

12. Prove each of the following for positive a and b:

a. $\sqrt{\frac{a^2+b^2}{2}} \geq \frac{a+b}{2}$ b. $\sqrt{ab} \geq \frac{2ab}{a+b}$

13. a. Extend this table to confirm the pattern and then express the relationship in if-then form.

$$2^2 - 1^2 = 3$$
$$3^2 - 2^2 = 5$$
$$4^2 - 3^2 = 7$$
$$5^2 - 4^2 = 9$$

b. Prove your relationship true.

14. Consider the statement:

If points A, B, and C are collinear, then B is the midpoint of $\overline{AC}$.

Is this statement true or false? If it is false, use an example to demonstrate that. Then write the converse and contrapositive of the statement, identifying each one, and state whether each is true or false. If any are false, show that by example.

15. Consider the statement:

A square is a quadrilateral with 4 right angles.

Write this statement in if-then form. Is the statement true or false? If it is false, use an example to show that. Then write the converse and contrapositive, identifying each one, and state whether each is true or false. If any are false, show that by example.

16. Consider the following four conditional statements.

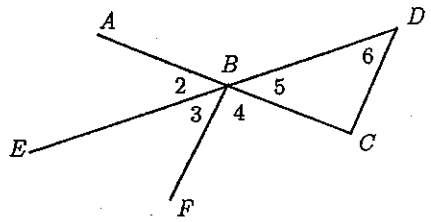
- If it is raining, then I'll get wet.
- If the sun is shining, then I won't get wet.
- If I'm in the middle of the Sahara Desert, then the sun is shining.
- If I need a camel for transportation, then I'm in the middle of the Sahara Desert.

a. Based on the logic of all four of the bullets, find a single conditional statement that uses all four of these "bullets."

b. What can you conclude if you are given the additional statement
- I need a camel for transportation.?

Exercises 1.4

Using the diagram at the right where B is the midpoint of $\overline{AC}$, write the reason that justifies each statement in problems 1–8.



1. $AB + BC = AC$

2. $AB = BC$

3. $\angle 2 \cong \angle 5$

4. $m\angle 2 + m\angle EBC = 180$

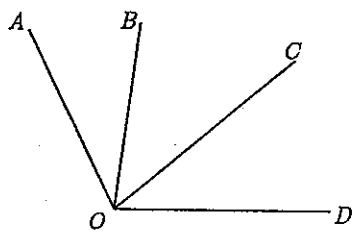
5. If $m\angle 5 + m\angle 6 = 90$, then $\angle 5$ and $\angle 6$ are complementary.

6. $AB = \frac{1}{2}(AC)$

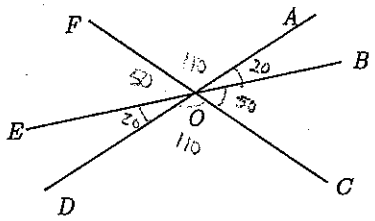
7. If $m\angle 2 = m\angle 3$, then $\overline{EB}$ bisects $\angle ABF$.

8. If $\angle FBC$ is a right angle, then $m\angle FBC = 90$.

9. If $m\angle COD = 40$, $m\angle AOD = 110$, and $\overline{OB}$ bisects $\angle AOC$, find $m\angle BOC$.



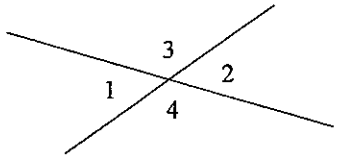
10. If $m\angle AOB = 20$ and $m\angle COE = 130$, find $m\angle AOC$.



11. a. If $m\angle 1 = 7x - 2$ and $m\angle 2 = 3x + 18$, find $m\angle 1$.

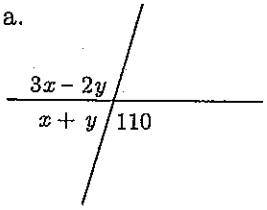
    b. If $m\angle 3 = x^2 - x$ and $m\angle 4 = 9x + 11$, find $m\angle 1$.

    c. If $m\angle 1 = 5y - 2$, $m\angle 2 = 3x + 4$, and $m\angle 3 = 5x$, determine x and y.

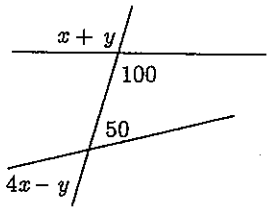


12. Find x and y in each of the following.

a.



b.



13. If the measure of the complement of an angle is 6 less than half the measure of the angle, find the measure of the angle.

14. If the measure of the supplement of an angle is 12 more than twice the measure of the angle, find the measure of the angle.

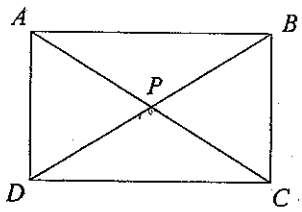
15. If the measure of the supplement of an angle is 6 more than three times the measure of the complement of the angle, find the measure of the angle.

16. If three times the measure of the complement of an angle is 20 less than the measure of the supplement of the angle, find the measure of the angle.

17. Can the measure of the complement of an angle equal half the measure of the supplement? Explain your answer.

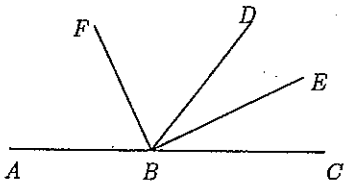
18. Given: $AP = PD$ and $CP = BP$

    Prove: $AC = BD$



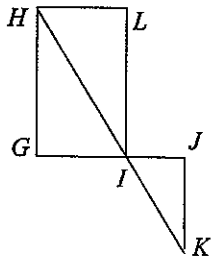
19. Given: $\overrightarrow{BF}$ bisects $\angle ABD$.
    $\overrightarrow{BE}$ bisects $\angle CBD$.

    Prove: $\angle FBD$ and $\angle EBD$ are complementary.



Exercises 1.5

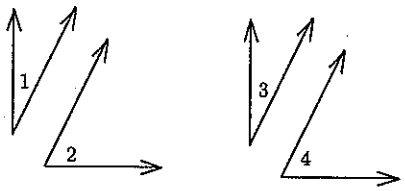
1. Write yes or no after each statement depending on whether the statement can be assumed from the diagram or not.



   a. H, I, and K are collinear.
   b. $\angle HGI$ and $\angle IJK$ are right angles.
   c. H, G, and K are non-collinear.
   d. $\angle GHI \cong \angle K$
   e. I is between G and J.
   f. $HGIL$ is a rectangle.

2. Supply the reasons in this proof of **Theorem 1.5**.

   If two angles are complements of congruent angles or of the same angle, then the two angles are congruent.



Given: $\angle 1$ and $\angle 2$ are complementary.
$\angle 3$ and $\angle 4$ are complementary.
$m\angle 2 = m\angle 4$

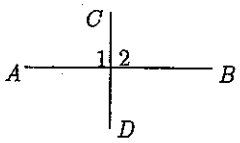
Prove: $m\angle 1 = m\angle 3$

Proof:

	Statements	Reasons
1.	$\angle 1$ and $\angle 2$ are complementary. $\angle 3$ and $\angle 4$ are complementary.	1.
2.	$m\angle 1 + m\angle 2 = 90$ $m\angle 3 + m\angle 4 = 90$	2.
3.	$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3.
4.	$m\angle 2 = m\angle 4$	4.
5.	$m\angle 1 = m\angle 3$	5.

3. Supply the reasons in this proof of **Theorem 1.6**.

   If two lines are perpendicular, then the adjacent angles they form are congruent.



Given: $\overline{AB} \perp \overline{CD}$

Prove: $m\angle 1 = m\angle 2$

Proof:

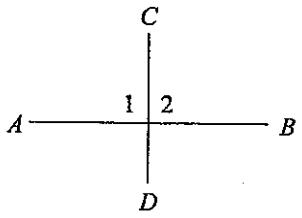
	Statements	Reasons
1.	$\overline{AB} \perp \overline{CD}$	1.
2.	$\angle 1$ and $\angle 2$ are right angles.	2.
3.	$m\angle 1 = 90$, $m\angle 2 = 90$	3.
4.	$m\angle 1 = m\angle 2$	4.

4. Supply the reasons in this proof of **Theorem 1.7**

If two lines form congruent adjacent angles, then the lines are perpendicular.

Given: $m\angle 1 = m\angle 2$

Prove: $\overline{AB} \perp \overline{CD}$



Proof:

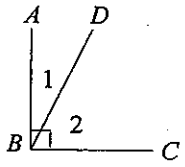
	Statements	Reasons
1.	$m\angle 1 = m\angle 2$	1.
2.	$m\angle 1 + m\angle 2 = 180$	2.
2.5	$m\angle 1 + m\angle 1 = 180$	2.5
3.	$2m\angle 1 = 180$	3.
4.	$m\angle 1 = 90$	4.
5.	$\angle 1$ is a right angle.	5.
6.	$\overline{AB} \perp \overline{CD}$	6.

5. Supply the reasons in this proof of **Theorem 1.8**

If the exterior sides of two adjacent, acute angles are perpendicular, then the adjacent angles are complementary.

Given: $\overline{AB} \perp \overline{BC}$

Prove: $\angle 1$ and $\angle 2$ are complementary.

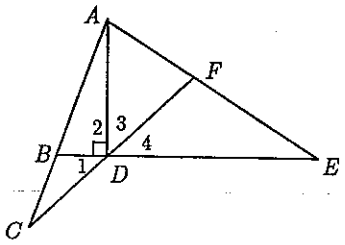


Proof:

	Statements	Reasons
1.	$\overline{AB} \perp \overline{BC}$	1.
2.	$\angle ABC$ is a right angle.	2.
3.	$m\angle ABC = 90$	3.
4.	$m\angle 1 + m\angle 2 = m\angle ABC$	4.
5.	$m\angle 1 + m\angle 2 = 90$	5.
6.	$\angle 1$ and $\angle 2$ are complementary.	6.

6. Write a proof for **Theorem 1.9**: All right angles are congruent.

Using the diagram at the right where $\overline{AD} \perp \overline{BE}$, D is the midpoint of $\overline{CF}$, $m\angle ADE = 90$, and $\overrightarrow{DF}$ bisects $\angle ADE$, write the reason, definition, postulate, or theorem that justifies each statement in problems 7–15.



7. $m\angle 3 = m\angle 4$

8. $BD + DE = BE$

9. $\angle ADC$ and $\angle ADF$ are supplementary.

10. $\angle 3$ and $\angle 4$ are complementary.

11. $CD = DF$

12. $m\angle CDB + m\angle BDA = m\angle CDA$

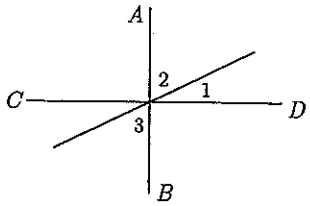
13. $\angle 1 \cong \angle 4$

14. $\angle BDA \cong \angle EDA$

15. If $m\angle 1 = m\angle 4$, then $m\angle 1 + m\angle 2 = m\angle 4 + m\angle 2$.

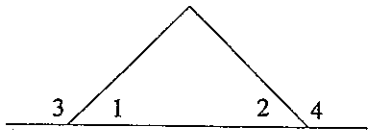
16. Given: $\overline{AB} \perp \overline{CD}$

Prove: $\angle 1$ and $\angle 3$ are complementary.



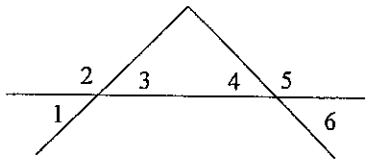
17. Given: $\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$



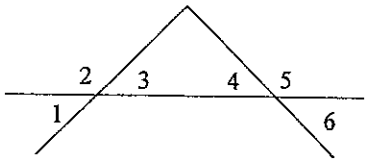
18. Given: $\angle 1 \cong \angle 4$

Prove: $\angle 1 \cong \angle 6$



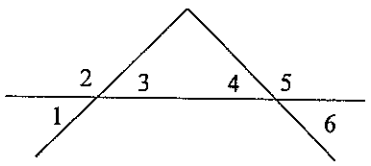
19. Given: $\angle 1 \cong \angle 6$

Prove: $\angle 3 \cong \angle 4$



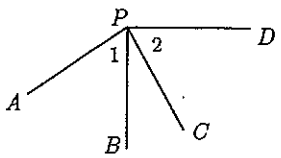
20. Given: $\angle 2 \cong \angle 5$

Prove: $\angle 1 \cong \angle 3 \cong \angle 4 \cong \angle 6$



21. Given: $\overline{AP} \perp \overline{CP}$
$\overline{BP} \perp \overline{DP}$

Prove: $\angle 1 \cong \angle 2$



Chapter 1 Review Exercises

1. Give the reasons for the following proof.

Given: $AC = BD$

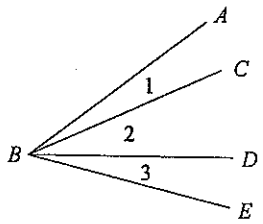
Prove: $AB = CD$

Statements	Reasons
1. $AC = BD$	1.
2. $AB + BC = AC,\ BC + CD = BD$	2.
3. $AB + BC = BC + CD$	3.
4. $BC = BC$	4.
5. $AB = CD$	5.

2. Write the reasons for the following proof.

Given: $m\angle ABD = m\angle EBC$

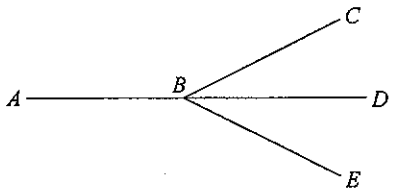
Prove: $m\angle 1 = m\angle 3$



Statements	Reasons
1. $m\angle ABD = m\angle EBC$	1.
2. $m\angle ABD = m\angle 1 + m\angle 2$	2.
3. $m\angle EBC = m\angle 3 + m\angle 2$	3.
4. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	4.
5. $m\angle 1 = m\angle 3$	5.

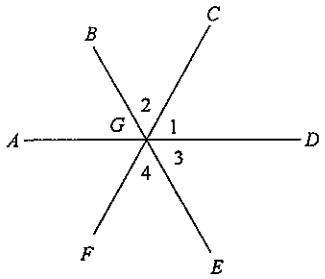
3. Given: $m\angle CBD = m\angle EBD$.

Prove: $m\angle ABC = m\angle ABE$



4. Given: $m\angle 1 = m\angle 3$

Prove: $m\angle BGD = m\angle FGD$



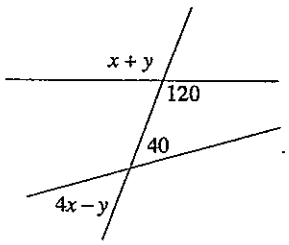
5. Is the following true or false? If $m\angle ABC = m\angle DBC$, then $\overrightarrow{BC}$ bisects $\angle ABD$. If false, explain why using a diagram.

6. If $m\angle LMN = 48$ and $m\angle LMQ = 37$, determine all possible measures for $\angle NMQ$.

7. Points A, B, and C are collinear with one of the points being the midpoint of the segment determined by the other two. $AB = x+7$ while $BC = 3x-3$. If the length of the whole segment is 48, then which point is the midpoint?

8. The points A, B, C, D, and E are arranged (in that order) along a line. In addition, $AB : BC : CD : DE = 1 : 2 : 3 : 4$. If $BD = 10$, then how long is $\overline{AB}$?

9. Determine x and y.



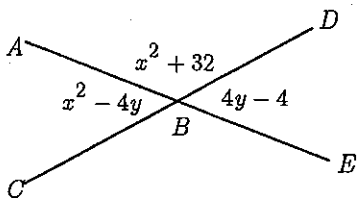
10. If an angle whose measure is $11x - 23$ is acute, determine the number of possible integral values for x.

11. If $AB = x^2 - 3x - 4$, $BC = 3x + 3$, and B is the midpoint of $\overline{AC}$, find the length of $\overline{AC}$.

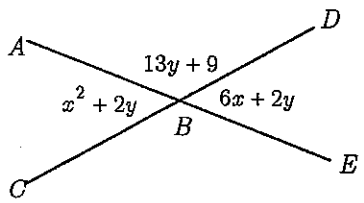
12. If five times the measure of the complement of an angle equals the measure of the supplement of the angle, find the measure of the angle.

13. If the ratio of the measure of the supplement of an angle to the measure of the complement of the same angle is 46:1, determine the measure of the angle.

14. Find all possible measures of $\angle ABC$.



15. Find all possible measures of $\angle ABC$.

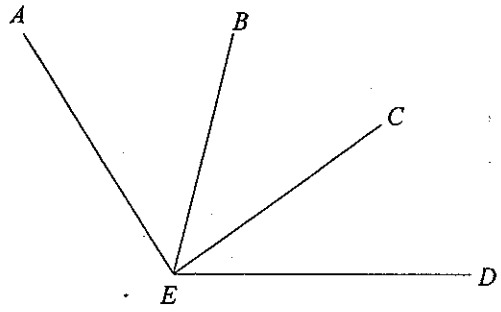


16. If $m\angle ABC = 2x + 45$, $\angle ABC$ is obtuse, $m\angle DEF = 3x - 40$ and $\angle DEF$ is acute, then find the number of integer values that x can take on.

17. The length of $\overline{AC}$ is 45. If B lies on $\overline{AC}$ such that $AB = 3x + 2$, $BC = 2y + 1$, and x and y are positive integers, determine all ordered pairs (x, y).

18. $\overrightarrow{BD}$ bisects $\angle ABC$, $m\angle ABD = 10x - 8$, and $m\angle DBC = 4y + 4$. Determine all ordered pairs of positive integers (x, y) satisfying the conditions of this problem.

19. If $\overrightarrow{EB}$ and $\overrightarrow{EC}$ trisect $\angle AED$ and $m\angle AEB = 2x + 2y$, $m\angle BEC = 3x + \frac{y}{2}$, and $m\angle CED = 20 + \frac{5y}{2}$, determine the values of x, y, and $m\angle AED$.



20. If the ratio of the measure of the complement of an angle to the measure of the supplement of the angle is greater than one-third, what inequality is satisfied by the measure of the angle?

21. If twice the measure of the complement of an angle is 30 less than the measure of the supplement of the angle, find the measure of the angle.

22. If the ratio of the measure of the complement of an angle to the measure of the supplement of the angle is less than one-tenth, determine the number of integer values that the measure of the angle can equal.

23. In triangle ABC, D lies on $\overline{AC}$ such that $\overrightarrow{BD}$ bisects $\angle ABC$. Also, $m\angle DBC = m\angle C$. If $m\angle ABD = 3x - y$, $m\angle DBC = 4y + 1$, and $m\angle C = 41 - x$, determine $m\angle C$.

24. The measures of two angles are in the ratio of 2 : 1. The measure of the larger one is 20 more than half the difference between the measures of the two angles. Find the measures of the two angles.

25. If $AB = x^2 - 4x + 3$, $BC = 2x - 1$, and B is the midpoint of $\overline{AC}$, find all possible lengths of $\overline{AC}$.

26. The diagrams below show a circle being divided into as many parts as possible when 2 or 3 points on the circumference are connected. Draw diagrams when the number of points is 4 and 5. Express the number of regions in terms of the number of points by an if-then statement. For fun you might try to draw the diagram when the number of points is 6.